

A formal exploration of **Nominal Kleene Algebra**

*41st International Symposium on
Mathematical Foundations of Computer Science*

Paul Brunet and Damien Pous
Équipe Plume – Univ Lyon, UCB Lyon 1, CNRS, ENS de Lyon, LIP

August 22-26 2016

Introduction

$x \leftarrow 1; \text{choice}(y \leftarrow x | y \leftarrow 0)$

$\text{choice}(x \leftarrow 1; y \leftarrow x | x \leftarrow 1; y \leftarrow 0)$

Introduction

$x \leftarrow 1; \text{choice}(y \leftarrow x | y \leftarrow 0)$

\downarrow
 $a \cdot (b + c)$

$\text{choice}(x \leftarrow 1; y \leftarrow x | x \leftarrow 1; y \leftarrow 0)$

\downarrow
 $(a \cdot b) + (a \cdot c)$

Introduction

$x \leftarrow 1; \text{choice}(y \leftarrow x | y \leftarrow 0)$ $\text{choice}(x \leftarrow 1; y \leftarrow x | x \leftarrow 1; y \leftarrow 0)$

$KA \quad \vdash \quad a \cdot (b + c) \quad = \quad (a \cdot b) + (a \cdot c)$

Introduction

$$x \leftarrow 1; \text{choice}(y \leftarrow x | y \leftarrow 0) \equiv \text{choice}(x \leftarrow 1; y \leftarrow x | x \leftarrow 1; y \leftarrow 0)$$

$$KA \quad \vdash \quad a \cdot (b + c) = (a \cdot b) + (a \cdot c)$$

Introduction

$$x \leftarrow 1; \text{choice}(y \leftarrow x | y \leftarrow 0) \equiv \text{choice}(x \leftarrow 1; y \leftarrow x | x \leftarrow 1; y \leftarrow 0)$$

$$KA \quad \vdash \quad a \cdot (b + c) \quad = \quad (a \cdot b) + (a \cdot c)$$

Fundamental Theorem of Kleene Algebra

$\forall e, f \in \text{Reg}\langle \Sigma \rangle,$

$$\text{Rel} \models e = f \quad \Leftrightarrow \quad KA \vdash e = f \quad \Leftrightarrow \quad L(e) = L(f)$$

Towards Nominal Kleene Algebra

A simple swap program

$$t \leftarrow x \quad ; \quad x \leftarrow y \quad ; \quad y \leftarrow t$$

Towards Nominal Kleene Algebra

A simple swap program

$t \leftarrow x$; $x \leftarrow y$; $y \leftarrow t$

tx

| | | | |
|-----|-------|-----|-------|
| t | v_1 | t | v_2 |
| x | v_2 | x | v_2 |
| y | v_3 | y | v_3 |

xy

| | | | |
|-----|-------|-----|-------|
| t | v_1 | t | v_1 |
| x | v_2 | x | v_3 |
| y | v_3 | y | v_3 |

yt

| | | | |
|-----|-------|-----|-------|
| t | v_1 | t | v_1 |
| x | v_2 | x | v_2 |
| y | v_3 | y | v_1 |

Towards Nominal Kleene Algebra

A simple swap program

$t \leftarrow x \quad ; \quad x \leftarrow y \quad ; \quad y \leftarrow t$

$tx \quad \circ \quad xy \quad \circ \quad yt$

| | |
|-----|-------|
| t | v_1 |
| x | v_2 |
| y | v_3 |

| | |
|-----|-------|
| t | v_2 |
| x | v_3 |
| y | v_1 |

Towards Nominal Kleene Algebra

A simple swap program

`local var t in {` $t \leftarrow x$; $x \leftarrow y$; $y \leftarrow t$ `}`
 tx \circ xy \circ yt

| | |
|-----|-------|
| t | v_1 |
| x | v_2 |
| y | v_3 |

| | |
|-----|-------|
| t | v_2 |
| x | v_3 |
| y | v_1 |

Towards Nominal Kleene Algebra

A simple swap program

`local var t in { $t \leftarrow x$; $x \leftarrow y$; $y \leftarrow t$ }`
`($\nu t.$ tx \circ xy \circ yt)`

| | |
|-----|-------|
| x | v_2 |
| y | v_3 |

| | |
|-----|-------|
| t | v_1 |
| x | v_2 |
| y | v_3 |

| | |
|-----|-------|
| t | v_2 |
| x | v_3 |
| y | v_2 |

| | |
|-----|-------|
| x | v_3 |
| y | v_2 |

Outline

Introduction

Nominal Kleene Algebra

Variations around Nominal Kleene Algebra

Results

Outline

Introduction

Nominal Kleene Algebra

Variations around Nominal Kleene Algebra

Results

Nominal set

\mathcal{A} an infinite set of atoms. $a, b, \dots \in \mathcal{A}$.

$() , (a\ b) , \pi , \dots \in \mathfrak{S}$ the set of finitely supported permutations over \mathcal{A} .

Nominal set

\mathcal{A} an infinite set of atoms. $a, b, \dots \in \mathcal{A}$.

$(\), (a\ b), \pi, \dots \in \mathfrak{S}$ the set of finitely supported permutations over \mathcal{A} .

X is a set and \bowtie is a map $\mathfrak{S} \times X \rightarrow X$. X is a **nominal set** over \mathcal{A} if:

▶ \bowtie is a **group action**:

$$\pi \bowtie (\pi' \bowtie x) = (\pi \circ \pi') \bowtie x;$$

Nominal set

\mathcal{A} an infinite set of atoms. $a, b, \dots \in \mathcal{A}$.

$() , (a b) , \pi , \dots \in \mathfrak{S}$ the set of finitely supported permutations over \mathcal{A} .

X is a set and \bowtie is a map $\mathfrak{S} \times X \rightarrow X$. X is a **nominal set** over \mathcal{A} if:

▶ \bowtie is a **group action**:

$$\pi \bowtie (\pi' \bowtie x) = (\pi \circ \pi') \bowtie x;$$

▶ every $x \in X$ has a **finite support** A :

$$A \subset \mathcal{A} \text{ is finite and } \forall \pi \in \mathfrak{S}, (\forall a \in A, \pi(a) = a) \Rightarrow \pi \bowtie x = x.$$

Nominal set

\mathcal{A} an infinite set of atoms. $a, b, \dots \in \mathcal{A}$.

$() , (a b) , \pi , \dots \in \mathfrak{S}$ the set of finitely supported permutations over \mathcal{A} .

X is a set and \bowtie is a map $\mathfrak{S} \times X \rightarrow X$. X is a **nominal set** over \mathcal{A} if:

▶ \bowtie is a **group action**:

$$\pi \bowtie (\pi' \bowtie x) = (\pi \circ \pi') \bowtie x;$$

▶ every $x \in X$ has a **finite support** A :

$$A \subset \mathcal{A} \text{ is finite and } \forall \pi \in \mathfrak{S}, (\forall a \in A, \pi(a) = a) \Rightarrow \pi \bowtie x = x.$$

$a \# x$ means a is not in the support of x .

Nominal Kleene Algebra

$$\mathcal{K} = \langle K, 0, 1, +, \cdot, *, \nu \rangle$$

(i) M. J. Gabbay and V. Ciancia. [Freshness and name-restriction in sets of traces with names.](#)
In *FoSSaCS 2011*. Springer, 2011

Nominal Kleene Algebra

$$\mathcal{K} = \langle K, 0, 1, +, \cdot, *, \nu \rangle$$

- ▶ $\langle K, 0, 1, +, \cdot, * \rangle$ is a **Kleene Algebra**;

⁽ⁱ⁾M. J. Gabbay and V. Ciancia. [Freshness and name-restriction in sets of traces with names.](#)
In *FoSSaCS 2011*. Springer, 2011

Nominal Kleene Algebra

$$\mathcal{K} = \langle K, 0, 1, +, \cdot, *, \nu \rangle$$

- ▶ $\langle K, 0, 1, +, \cdot, * \rangle$ is a **Kleene Algebra**;
- ▶ K is a **nominal set** over atoms \mathcal{A} ;

⁽ⁱ⁾M. J. Gabbay and V. Ciancia. [Freshness and name-restriction in sets of traces with names.](#)
In *FoSSaCS 2011*. Springer, 2011

Nominal Kleene Algebra

$$\mathcal{K} = \langle K, 0, 1, +, \cdot, *, \nu \rangle$$

- ▶ $\langle K, 0, 1, +, \cdot, * \rangle$ is a **Kleene Algebra**;
- ▶ K is a **nominal set** over atoms \mathcal{A} ;
- ▶ \mathcal{K} satisfies the **nominal axioms**⁽ⁱ⁾:

$$\nu a. (e + f) = \nu a. e + \nu a. f$$

$$\nu a. (\nu b. e) = \nu b. (\nu a. e)$$

$$a \# e \Rightarrow \nu a. e = e$$

$$a \# e \Rightarrow \nu b. e = \nu a. (a \ b) \bowtie e$$

$$a \# e \Rightarrow \nu a. f \cdot e = \nu a. (f \cdot e)$$

$$a \# e \Rightarrow e \cdot \nu a. f = \nu a. (e \cdot f)$$

⁽ⁱ⁾M. J. Gabbay and V. Ciancia. [Freshness and name-restriction in sets of traces with names.](#)
In *FoSSaCS 2011*. Springer, 2011

Nominal Kleene Algebra

$$\mathcal{K} = \langle K, 0, 1, +, \cdot, *, \nu \rangle$$

- ▶ $\langle K, 0, 1, +, \cdot, * \rangle$ is a **Kleene Algebra**;
- ▶ K is a **nominal set** over atoms \mathcal{A} ;
- ▶ \mathcal{K} satisfies the **nominal axioms**⁽ⁱ⁾:

$$\begin{array}{ll} \nu a. (e + f) = \nu a. e + \nu a. f & a \# e \Rightarrow \nu b. e = \nu a. (a \ b) \bowtie e \\ \nu a. (\nu b. e) = \nu b. (\nu a. e) & a \# e \Rightarrow \nu a. f \cdot e = \nu a. (f \cdot e) \\ a \# e \Rightarrow \nu a. e = e & a \# e \Rightarrow e \cdot \nu a. f = \nu a. (e \cdot f) \end{array}$$

- ▶ \mathcal{K} satisfies the **permutation axioms**:

$$\begin{array}{lll} \pi \bowtie 1 := 1 & \pi \bowtie (\nu_a. e) := \nu_{\pi(a)}. (\pi \bowtie e) & \pi \bowtie (e^*) := (\pi \bowtie e)^* \\ \pi \bowtie 0 := 0 & \pi \bowtie (e \cdot f) := \pi \bowtie e \cdot \pi \bowtie f & \pi \bowtie (e + f) := \pi \bowtie e + \pi \bowtie f \end{array}$$

⁽ⁱ⁾M. J. Gabbay and V. Ciancia. [Freshness and name-restriction in sets of traces with names.](#)
In *FoSSaCS 2011*. Springer, 2011

Completeness of NKA

$$e, f ::= 0 \mid 1 \mid a \mid e + f \mid e \cdot f \mid e^* \mid \nu a.e$$

(ii) D. Kozen, K. Mamouras, and A. Silva. [Completeness and incompleteness in Nominal Kleene Algebra](#).

In *RAMiCS 2015*. Springer, 2015

(iii) D. Kozen, K. Mamouras, D. Petrisan, and A. Silva. [Nominal Kleene Coalgebra](#).

In *ICALP 2015*. Springer, 2015

Completeness of NKA

$$e, f ::= 0 \mid 1 \mid a \mid e + f \mid e \cdot f \mid e^* \mid \nu a.e$$
$$NKA \models e = f \Leftrightarrow NKA \vdash e = f \Leftrightarrow L(e) = L(f)^{(ii)}$$

⁽ⁱⁱ⁾D. Kozen, K. Mamouras, and A. Silva. [Completeness and incompleteness in Nominal Kleene Algebra](#).

In *RAMiCS 2015*. Springer, 2015

⁽ⁱⁱⁱ⁾D. Kozen, K. Mamouras, D. Petrisan, and A. Silva. [Nominal Kleene Coalgebra](#).

In *ICALP 2015*. Springer, 2015

Completeness of NKA

$$e, f ::= 0 \mid 1 \mid a \mid e + f \mid e \cdot f \mid e^* \mid \nu a.e$$
$$NKA \models e = f \Leftrightarrow NKA \vdash e = f \Leftrightarrow L(e) = L(f)^{(ii)}$$

$L(e) = L(f)$ is **decidable**.⁽ⁱⁱⁱ⁾

⁽ⁱⁱ⁾D. Kozen, K. Mamouras, and A. Silva. [Completeness and incompleteness in Nominal Kleene Algebra](#).

In *RAMiCS 2015*. Springer, 2015

⁽ⁱⁱⁱ⁾D. Kozen, K. Mamouras, D. Petrisan, and A. Silva. [Nominal Kleene Coalgebra](#).

In *ICALP 2015*. Springer, 2015

Problem

$$e, f ::= 0 \mid 1 \mid a \mid e + f \mid e \cdot f \mid e^* \mid \nu a.e$$

Leaves of the expression = atoms.

Problem

$$e, f ::= 0 \mid 1 \mid a \mid e + f \mid e \cdot f \mid e^* \mid \nu a.e$$

Leaves of the expression = atoms.

⇒ Impossible to associate such an expression to a program:

$$x \leftarrow y \rightsquigarrow x?, y?$$

$$x \leftarrow 5 \neq x \leftarrow 2$$

Problem

$$e, f ::= 0 \mid 1 \mid a \mid e + f \mid e \cdot f \mid e^* \mid \nu a.e$$

Leaves of the expression = atoms.

⇒ Impossible to associate such an expression to a program:

$$x \leftarrow y \rightsquigarrow x?, y?$$

$$x \leftarrow 5 \neq x \leftarrow 2$$

⇒ We need to change the syntax of expressions, and adapt the axiomatisation.

Outline

Introduction

Nominal Kleene Algebra

Variations around Nominal Kleene Algebra

Results

Modified nominal expressions

$e, f ::= 0 \mid 1 \mid a \mid e+f \mid e \cdot f \mid e^* \mid \nu a.e$

Modified nominal expressions

$e, f ::= 0 \mid 1 \mid a \mid x \mid e+f \mid e \cdot f \mid e^* \mid \nu a.e$

$x, y, \dots \in X$ a nominal set over \mathcal{A} .

Modified nominal expressions

$e, f ::= 0 \mid 1 \mid a \mid x \mid e+f \mid e \cdot f \mid e^* \mid \nu a.e$

$b \# e \Rightarrow \nu a.e = \nu b.(a \ b) \bowtie e$

Modified nominal expressions

$e, f ::= 0 \mid 1 \mid a \mid x \mid e+f \mid e \cdot f \mid e^* \mid \nu a.e \mid \langle \pi \rangle e$

$b \# e \Rightarrow \nu a.e = \nu b.\langle (a \ b) \rangle e$

Modified nominal expressions

$e, f ::= 0 \mid 1 \mid a \mid x \mid e+f \mid e \cdot f \mid e^* \mid \nu a.e \mid \langle \pi \rangle e$

Presheaf presentation of nominal sets:

Modified nominal expressions

$e, f ::= 0 \mid 1 \mid a \mid x \mid e+f \mid e \cdot f \mid e^* \mid \nu a.e \mid \langle \pi \rangle e$

Presheaf presentation of nominal sets:

- ▶ every element of X has a type $A \in \mathcal{P}_f(\mathcal{A})$.

Modified nominal expressions

$e, f ::= 0 \mid 1 \mid a \mid x \mid e+f \mid e \cdot f \mid e^* \mid \nu a.e \mid \langle \pi \rangle e$

Presheaf presentation of nominal sets:

- ▶ every element of X has a type $A \in \mathcal{P}_f(\mathcal{A})$.
- ▶ $a \# x$ is replaced by $x : A$ and $a \notin A$.

Modified nominal expressions

$e, f ::= 0 \mid 1 \mid a \mid x \mid e+f \mid e \cdot f \mid e^* \mid \nu a.e \mid \langle \pi \rangle e$

Presheaf presentation of nominal sets:

- ▶ every element of X has a type $A \in \mathcal{P}_f(\mathcal{A})$.
- ▶ $a \# x$ is replaced by $x : A$ and $a \notin A$.
- ▶ $e + f$ is valid if e and f have the same type.

Modified nominal expressions

$e, f ::= 0 \mid 1 \mid id_A \mid \perp_A \mid a \mid x \mid e+f \mid e \cdot f \mid e^* \mid \nu a.e \mid \langle \pi \rangle e \mid wa.e$

Presheaf presentation of nominal sets:

- ▶ every element of X has a type $A \in \mathcal{P}_f(\mathcal{A})$.
- ▶ $a \# x$ is replaced by $x : A$ and $a \notin A$.
- ▶ $e + f$ is valid if e and f have the same type.

Modified nominal expressions

$e, f ::= 0 \mid 1 \mid id_A \mid \perp_A \mid a \mid x \mid e+f \mid e \cdot f \mid e^* \mid \nu a.e \mid \langle \pi \rangle e \mid wa.e$

Questions:

Modified nominal expressions

$e, f ::= 0 \mid 1 \mid id_A \mid \perp_A \mid a \mid x \mid e+f \mid e \cdot f \mid e^* \mid \nu a.e \mid \langle \pi \rangle e \mid wa.e$

Questions:

1. What axiomatisations should we give for these expressions?

Modified nominal expressions

$e, f ::= 0 \mid 1 \mid id_A \mid \perp_A \mid a \mid x \mid e+f \mid e \cdot f \mid e^* \mid \nu a.e \mid \langle \pi \rangle e \mid wa.e$

Questions:

1. What axiomatisations should we give for these expressions?
2. Are these modifications independent?

Modified nominal expressions

$e, f ::= 0 \mid 1 \mid id_A \mid \perp_A \mid a \mid x \mid e+f \mid e \cdot f \mid e^* \mid \nu a.e \mid \langle \pi \rangle e \mid wa.e$

Questions:

1. What axiomatisations should we give for these expressions?
2. Are these modifications independent?
3. Could we salvage some results of NKA?

A modular deduction system

$$\overline{Ax \vdash e = f}$$

A modular deduction system

$$\overline{Ax \vdash e = f}$$

- ▶ Structural rules:

$$\frac{Ax \vdash f = e}{Ax \vdash e = f}$$

$$\frac{Ax \vdash e = f \quad Ax \vdash f = g}{Ax \vdash e = f}$$

$$\frac{}{Ax \vdash e = f} \quad (e, f) \in Ax$$

A modular deduction system

$$\overline{Ax \vdash e = f}$$

- ▶ Structural rules:

$$\frac{Ax \vdash f = e}{Ax \vdash e = f}$$

$$\frac{Ax \vdash e = f \quad Ax \vdash f = g}{Ax \vdash e = g}$$

$$\frac{}{Ax \vdash e = f} \quad (e, f) \in Ax$$

- ▶ Congruence rules:

$$\frac{}{Ax \vdash 0 = 0}$$

$$\frac{}{Ax \vdash id_A = id_A}$$

$$\frac{Ax \vdash e = g \quad Ax \vdash f = h}{Ax \vdash e + f = g + h} \quad \dots$$

A modular deduction system

$$\overline{Ax \vdash e = f}$$

- ▶ Structural rules:

$$\frac{Ax \vdash f = e}{Ax \vdash e = f}$$

$$\frac{Ax \vdash e = f \quad Ax \vdash f = g}{Ax \vdash e = g}$$

$$\frac{}{Ax \vdash e = f} \quad (e, f) \in Ax$$

- ▶ Congruence rules:

$$\frac{}{Ax \vdash 0 = 0}$$

$$\frac{}{Ax \vdash id_A = id_A}$$

$$\frac{Ax \vdash e = g \quad Ax \vdash f = h}{Ax \vdash e + f = g + h} \quad \dots$$

- ▶ Constant-free Kleene Algebra axioms:

$$\frac{}{Ax \vdash e + f = f + e}$$

$$\frac{}{Ax \vdash e \cdot (f + g) = (e \cdot f) + (e \cdot g)}$$

$$\frac{Ax \vdash f + e \cdot g \leq g}{Ax \vdash e^* \cdot f \leq g} \quad \dots$$

Theories

Definition

A theory is a pair of a set of expressions and a set of axioms.

Theories

Definition

A theory is a pair of a set of expressions and a set of axioms.

Sets of expressions:

Theories

Definition

A theory is a pair of a set of expressions and a set of axioms.

Sets of expressions:

- ▶ atomic or nominal letters $\mathbb{E}\langle\mathcal{A}\rangle$ vs $\mathbb{E}\langle X\rangle$.

Theories

Definition

A theory is a pair of a set of expressions and a set of axioms.

Sets of expressions:

- ▶ atomic or nominal letters $\mathbb{E}\langle\mathcal{A}\rangle$ vs $\mathbb{E}\langle X\rangle$.
- ▶ syntactic or algebraic permutations \mathbb{E} vs \mathbb{C} .

Theories

Definition

A theory is a pair of a set of expressions and a set of axioms.

Sets of expressions:

- ▶ atomic or nominal letters $\mathbb{E}\langle\mathcal{A}\rangle$ vs $\mathbb{E}\langle X\rangle$.
- ▶ syntactic or algebraic permutations \mathbb{E} vs \mathbb{C} .
- ▶ typed or untyped expressions \mathbb{T} vs \mathbb{U} .

Theories

Definition

A theory is a pair of a set of expressions and a set of axioms.

Sets of expressions:

- ▶ atomic or nominal letters $\mathbb{E}\langle\mathcal{A}\rangle$ vs $\mathbb{E}\langle X\rangle$.
- ▶ syntactic or algebraic permutations \mathbb{E} vs \mathbb{C} .
- ▶ typed or untyped expressions \mathbb{T} vs \mathbb{U} .
- ▶ expressions with or without $0/\perp_A$ \mathbb{E} vs \mathbb{E}^+ .

Theories

Definition

A theory is a pair of a set of expressions and a set of axioms.

Sets of expressions: \Rightarrow 16 theories

- ▶ atomic or nominal letters $\mathbb{E}\langle\mathcal{A}\rangle$ vs $\mathbb{E}\langle X\rangle$.
- ▶ syntactic or algebraic permutations \mathbb{E} vs \mathbb{C} .
- ▶ typed or untyped expressions \mathbb{T} vs \mathbb{U} .
- ▶ expressions with or without $0/\perp_A$ \mathbb{E} vs \mathbb{E}^+ .

Ordering theories

How can two theories be compared?

Ordering theories

How can two theories be compared?

Expressiveness.

Ordering theories

How can two theories be compared?

Expressiveness.
Decidability.

Ordering theories

How can two theories be compared?

Expressiveness.
Decidability.

Embedding preorder

$(S, Ax) \preceq (S', Ax')$ if there is a function $\varphi : S \rightarrow S'$ such that $\forall e, f \in S$:

$$Ax \vdash e = f \Leftrightarrow Ax' \vdash \varphi e = \varphi f.$$

Ordering theories

How can two theories be compared?

Expressiveness.
Decidability.

Embedding preorder

$(S, Ax) \preceq (S', Ax')$ if there is a function $\varphi : S \rightarrow S'$ such that $\forall e, f \in S$:

$$Ax \vdash e = f \Leftrightarrow Ax' \vdash \varphi e = \varphi f.$$

$\mathbb{T} \vdash id_{\{a\}} \neq id_{\{a,b\}}$ but $\mathbb{U} \vdash 1 = 1$.

Ordering theories

How can two theories be compared?

Expressiveness.
Decidability.

Embedding preorder

$(S, Ax) \preceq (S', Ax')$ if there is a function $\varphi : S \rightarrow S'$ such that $\forall e, f \in S$:

$$Ax \vdash e = f \Leftrightarrow Ax' \vdash \varphi e = \varphi f.$$

$\mathbb{T} \vdash id_{\{a\}} \neq id_{\{a,b\}}$ but $\mathbb{U} \vdash 1 = 1$.

Reduction preorder

$(S, Ax) \ll (S', Ax')$ if for every pair $\langle e, f \rangle \in S \times S$ there is a pair $\langle e', f' \rangle \in S' \times S'$ such that

$$Ax \vdash e = f \Leftrightarrow Ax' \vdash e' = f'.$$

Outline

Introduction

Nominal Kleene Algebra

Variations around Nominal Kleene Algebra

Results

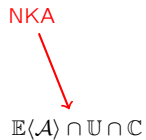
Results

NKA

\longrightarrow : embedding

\dashrightarrow : reduction

Results



————→ : embedding

-----> : reduction

Results

NKA




$$E\langle \mathcal{A} \rangle \cap U \cap C \overset{\longleftarrow}{\dashrightarrow} E^+\langle \mathcal{A} \rangle \cap U \cap C$$

\longrightarrow : embedding

\dashrightarrow : reduction

Results

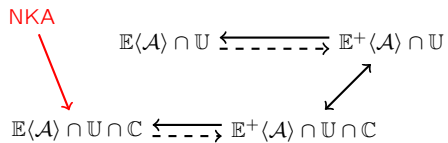
NKA

$$\mathbb{E}\langle \mathcal{A} \rangle \cap \mathbb{U} \begin{array}{c} \xrightarrow{\quad} \\ \xrightarrow{\quad\quad\quad} \end{array} \mathbb{E}^+\langle \mathcal{A} \rangle \cap \mathbb{U}$$

$$\mathbb{E}\langle \mathcal{A} \rangle \cap \mathbb{U} \cap \mathbb{C} \begin{array}{c} \xrightarrow{\quad} \\ \xrightarrow{\quad\quad\quad} \end{array} \mathbb{E}^+\langle \mathcal{A} \rangle \cap \mathbb{U} \cap \mathbb{C}$$

\longrightarrow : embedding

\dashrightarrow : reduction

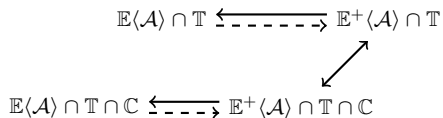
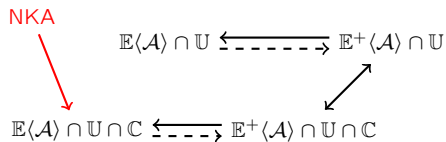
Results



\longrightarrow : embedding

\dashrightarrow : reduction

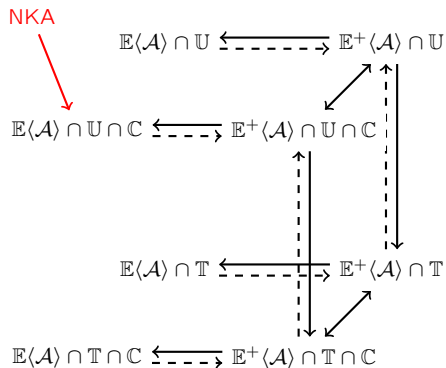
Results



\longrightarrow : embedding

\dashrightarrow : reduction

Results

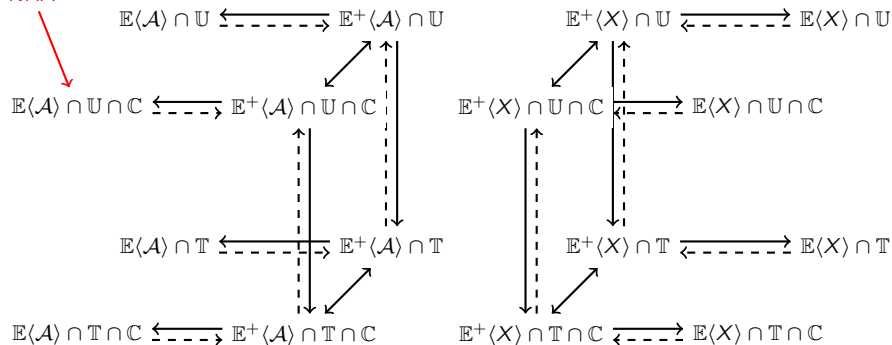


—————→ : embedding

-----→ : reduction

Results

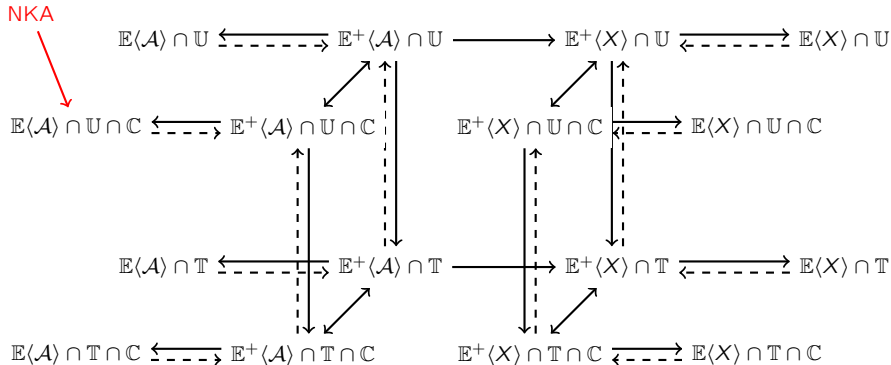
NKA



\longrightarrow : embedding

\dashrightarrow : reduction

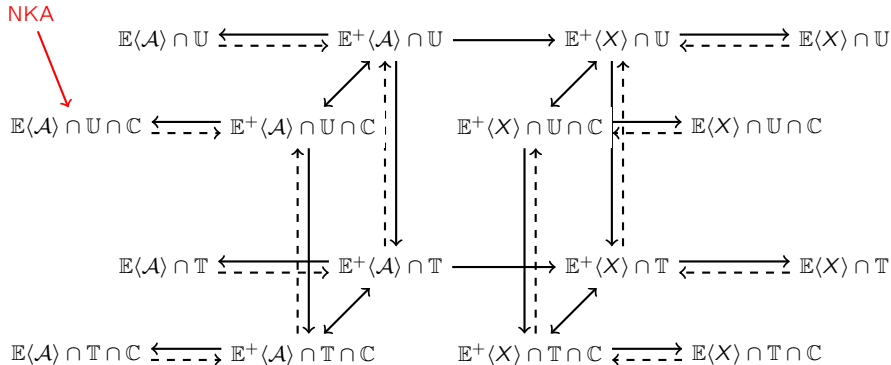
Results



\longrightarrow : embedding

\dashrightarrow : reduction

Results



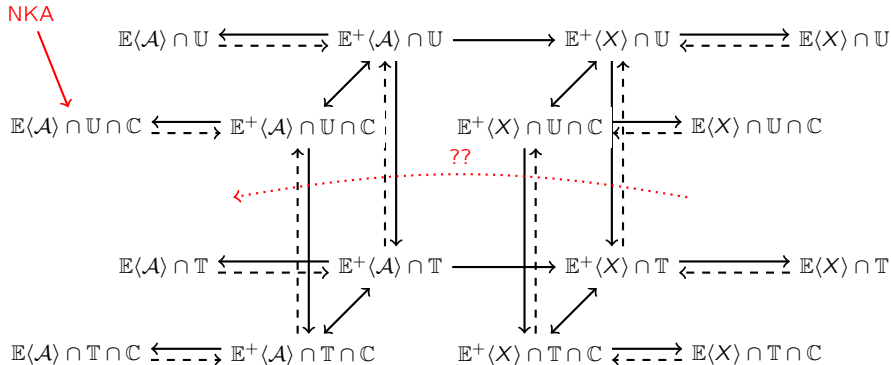
→ : embedding

⇔ : reduction

Proved using the proof assistant Coq (~ 9000 lines of proof script).

<http://perso.ens-lyon.fr/paul.brunet/nka>

Results



Proved using the proof assistant Coq (\sim 9000 lines of proof script).

<http://perso.ens-lyon.fr/paul.brunet/nka>

That's all folks!

Thank you!

See more at:

<http://perso.ens-lyon.fr/paul.brunet>

Outline

Introduction

Nominal Kleene Algebra

Variations around Nominal Kleene Algebra

Results