Pomsets with boxes

Protection, Separation, and Locality in Concurrent Kleene Algebra

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University College London

UCL

Concurrent Kleene Algebra

an algebraic framework to reason about concurrent programs

☞ Programs: $0 \mid 1 \mid a \in A \mid e \cdot f \mid e \parallel f \mid e + f \mid e^\star$

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abort execution

Concurrent Kleene Algebra

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skip

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abort execution

atomic action

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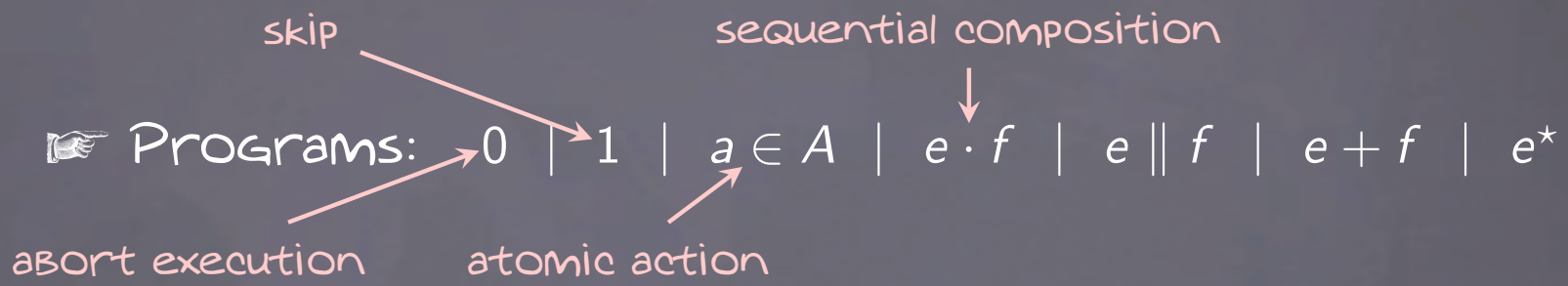

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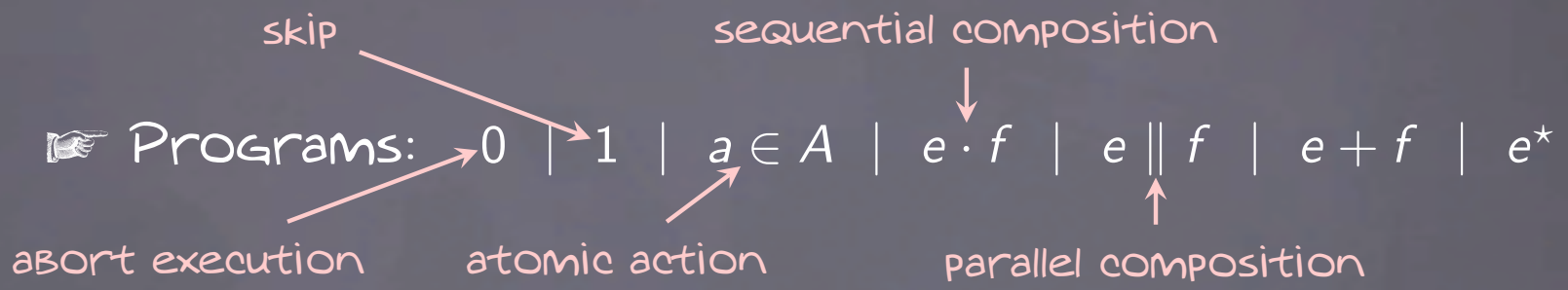

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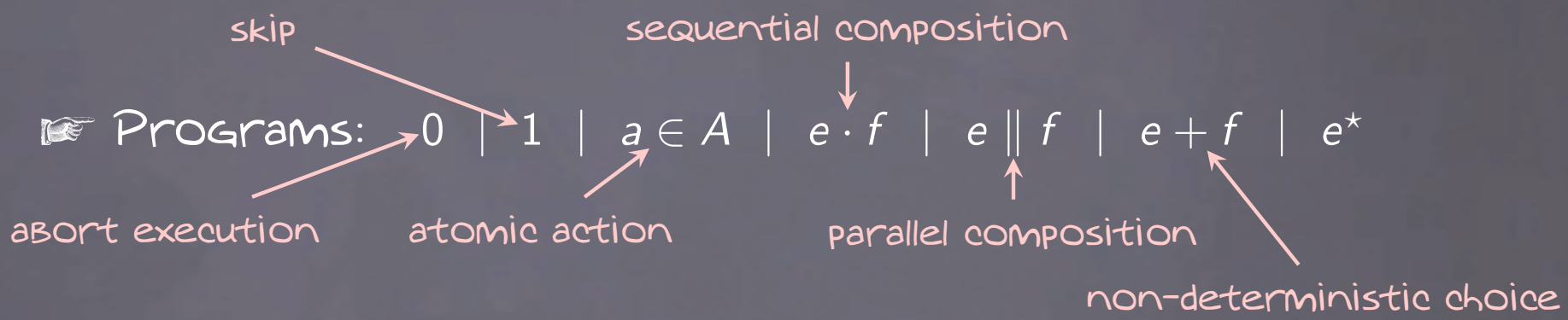

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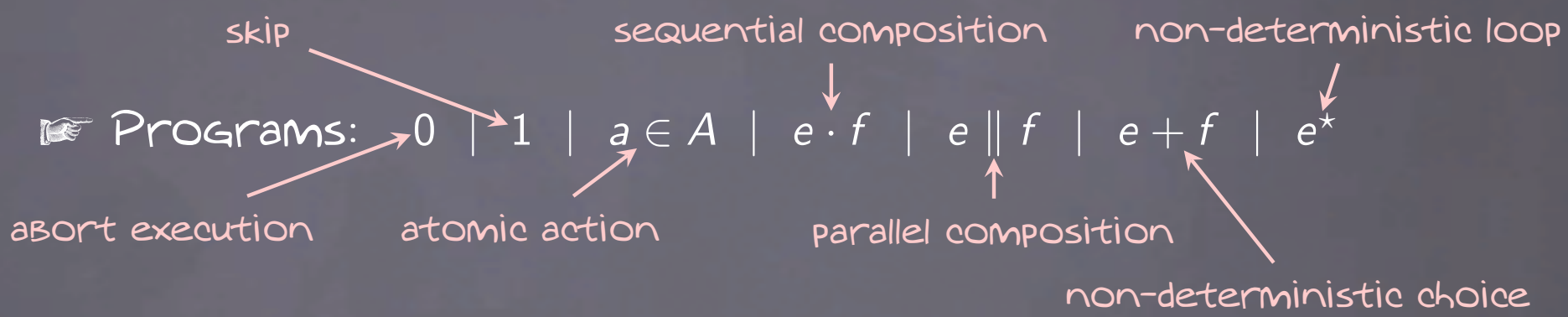
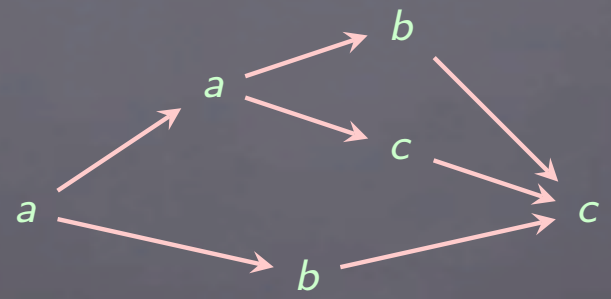


Concurrent Kleene Algebra

an algebraic framework to reason about concurrent programs

- Programs: $0 \mid 1 \mid a \in A \mid e \cdot f \mid e \parallel f \mid e + f \mid e^*$
- Semantics: sets of pomsets, i.e. partial traces



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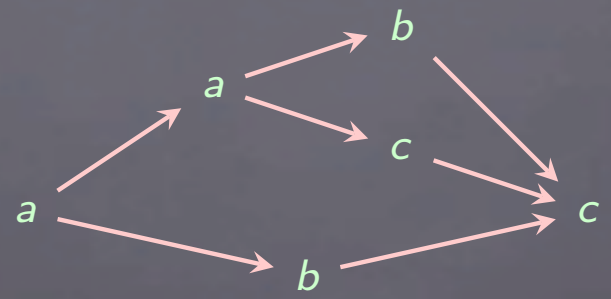


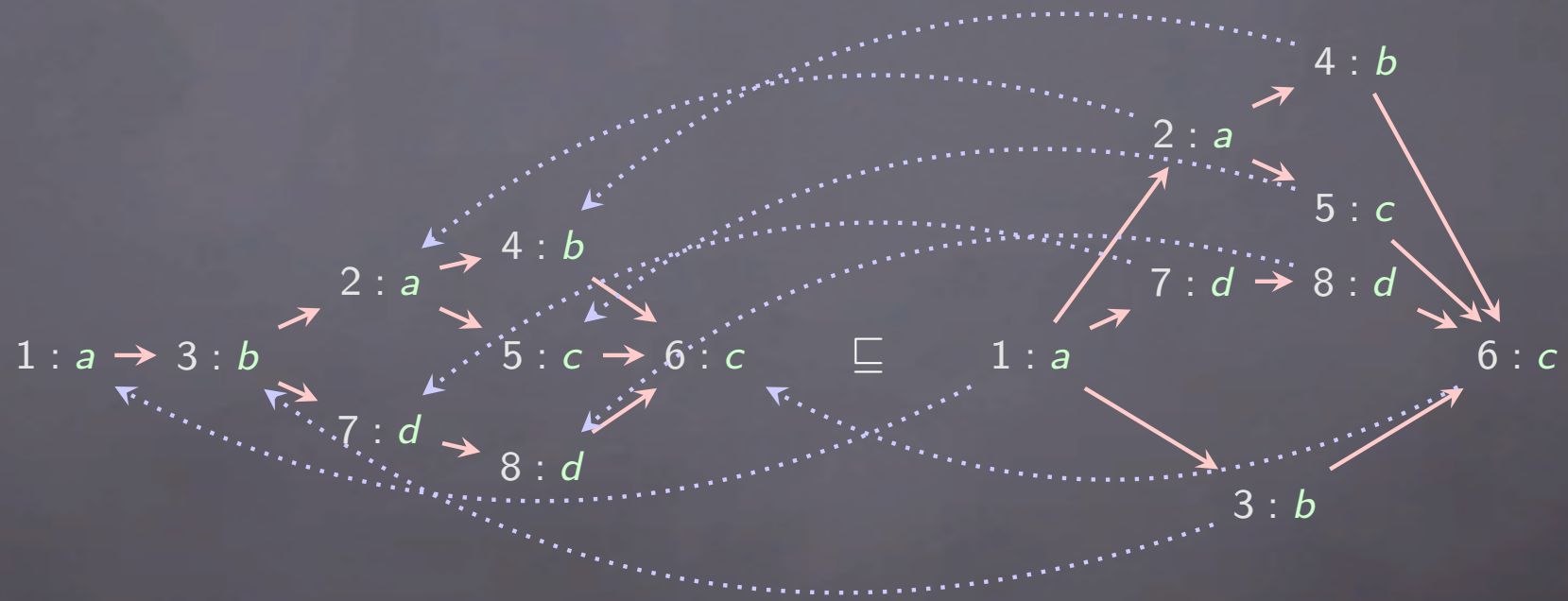
- Specifications: $[\![e]\!] \leq [\![f]\!]$.

  "All behaviours of the program e satisfy the property f."

Interleavings

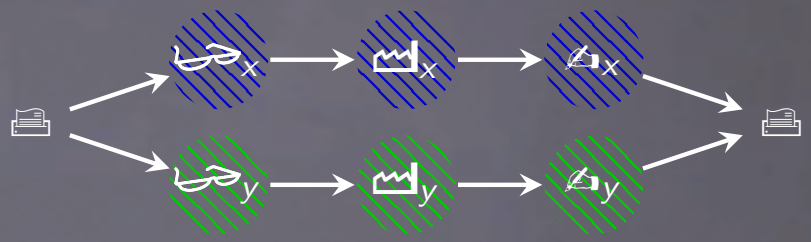
Interchange law

$$(a \parallel b) \cdot (c \parallel d) \leq (a \cdot c) \parallel (b \cdot d).$$



Mutual exclusion

```
print(counter);
        ||
x:=counter;  ||  y:=counter;
x:=x+1;      ||  y:=y+1;
counter:=x;  ||  counter:=y;
        ||
print(counter);
```



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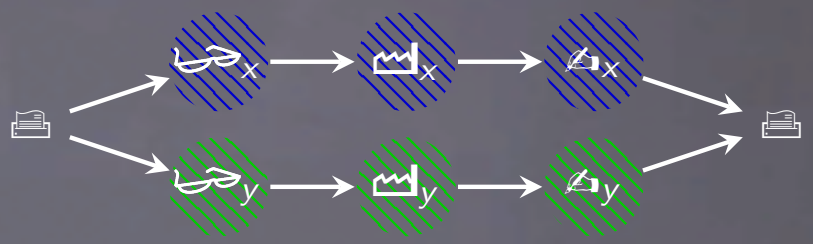

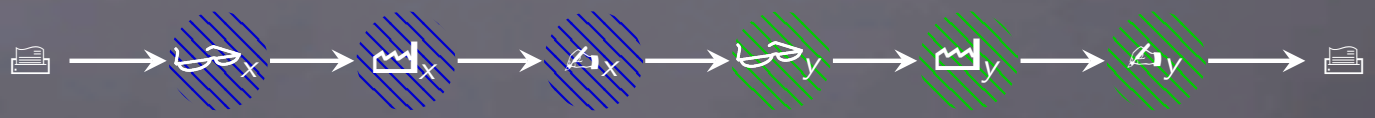

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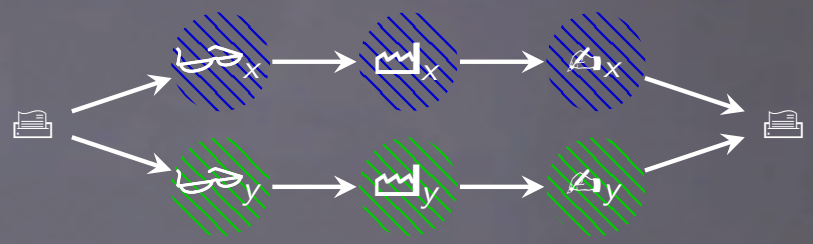

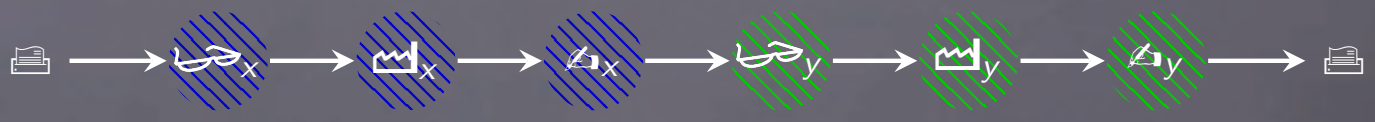

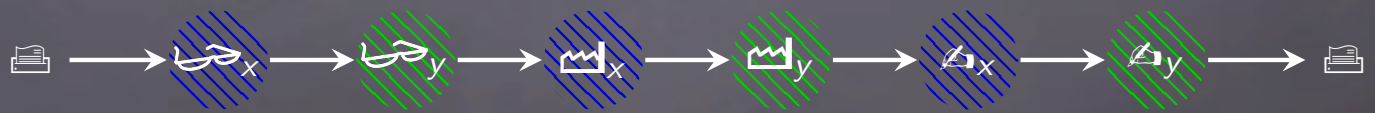

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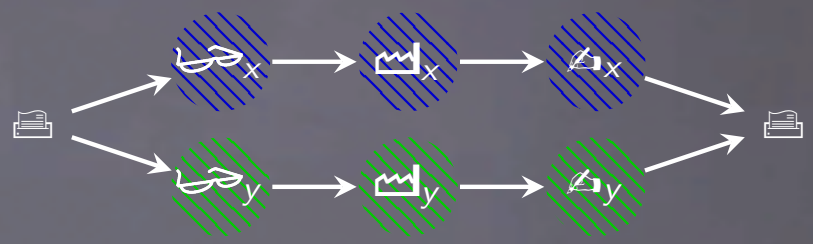

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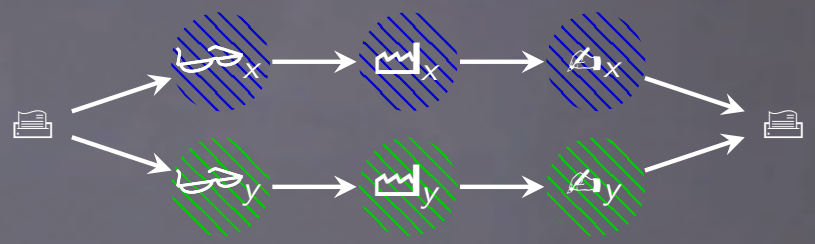

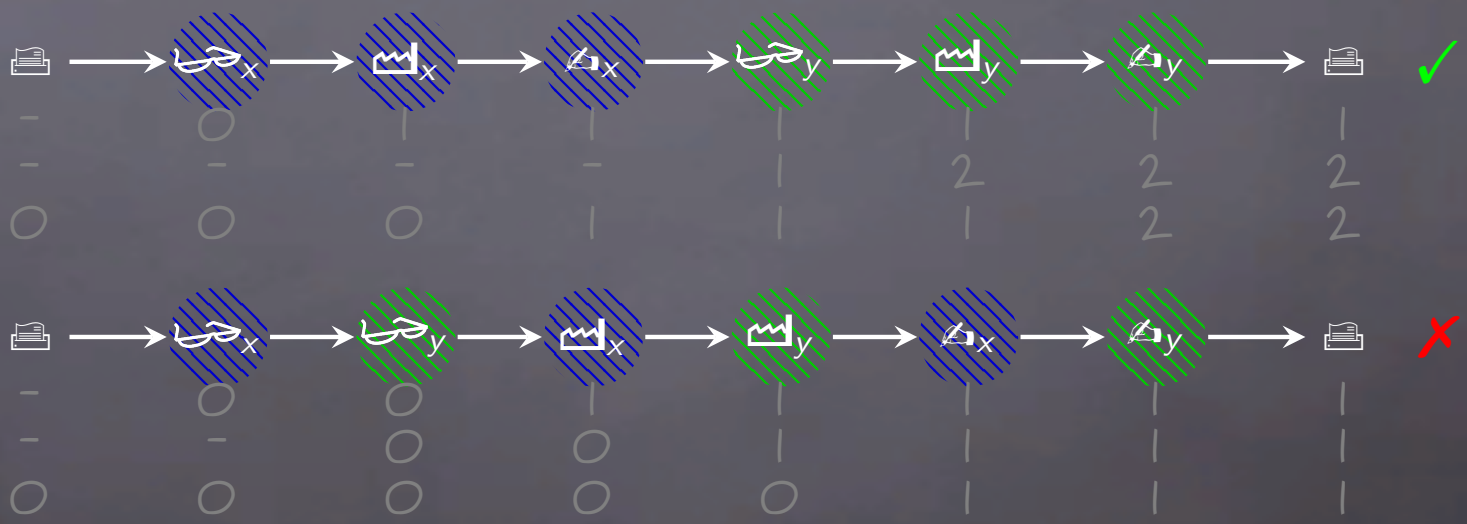

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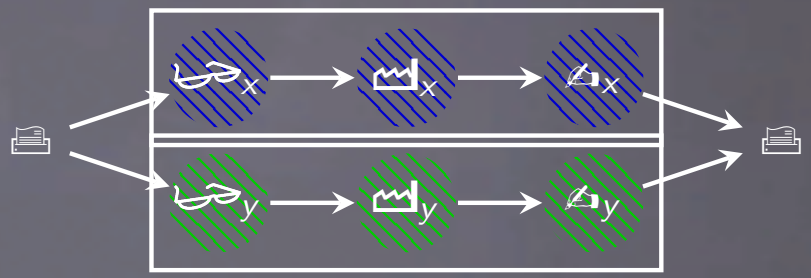

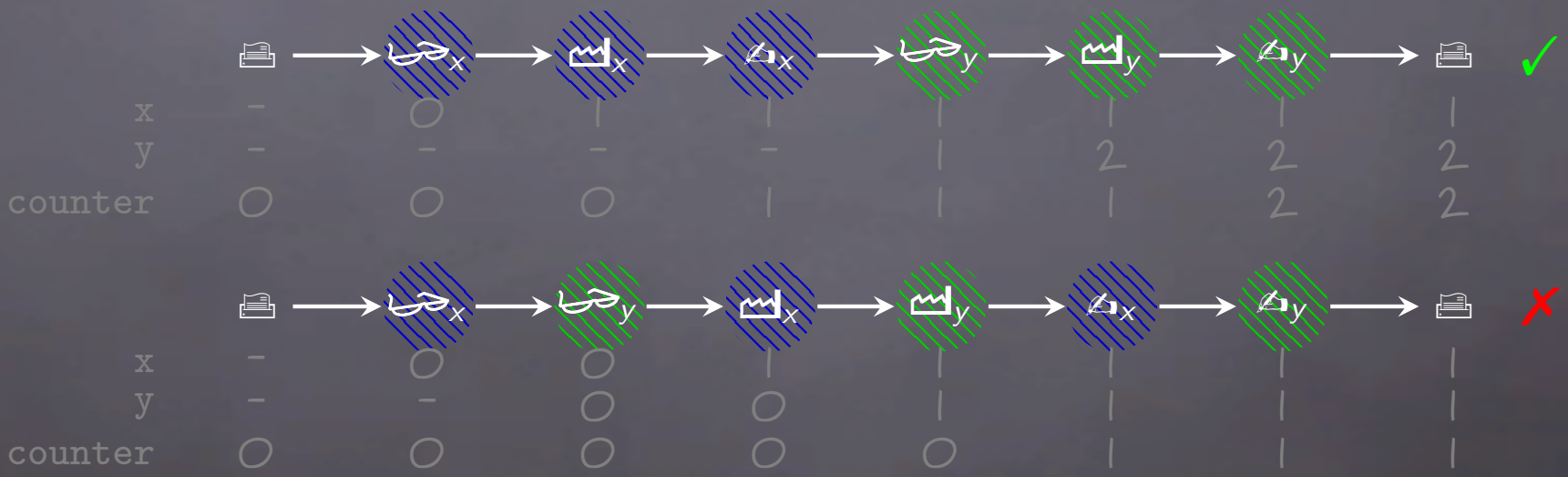

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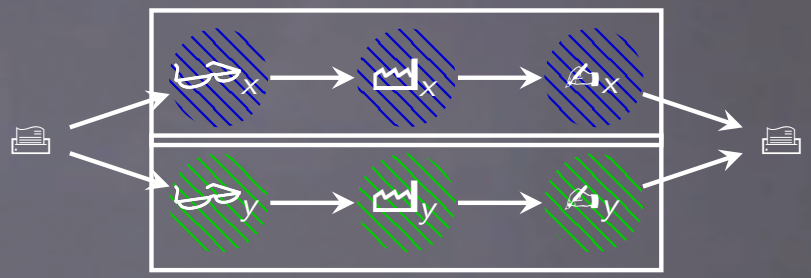

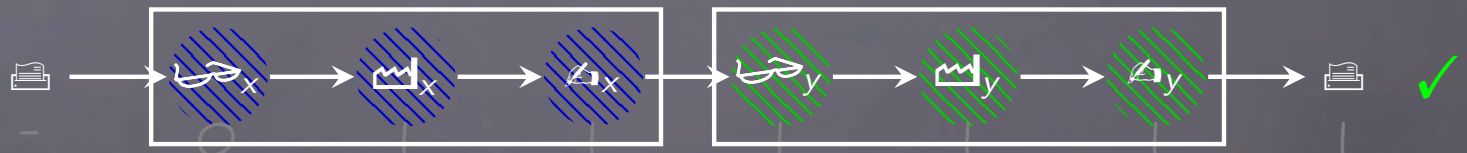

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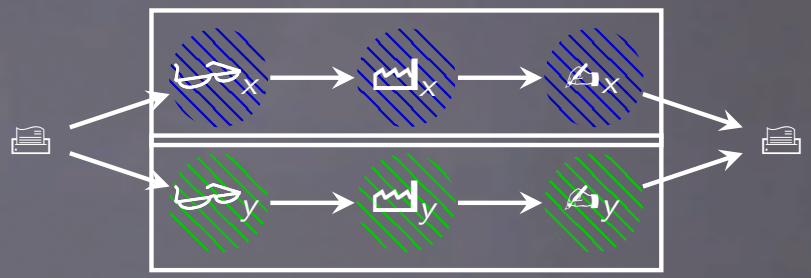
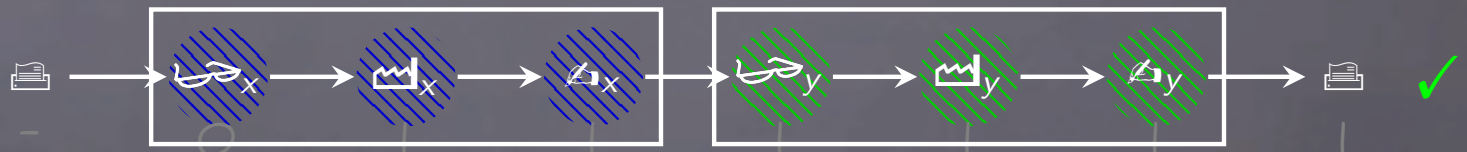
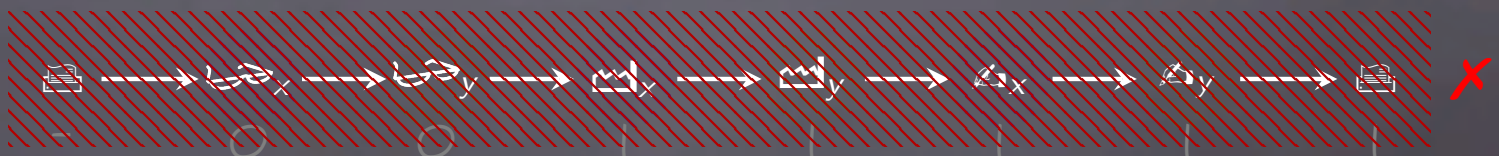
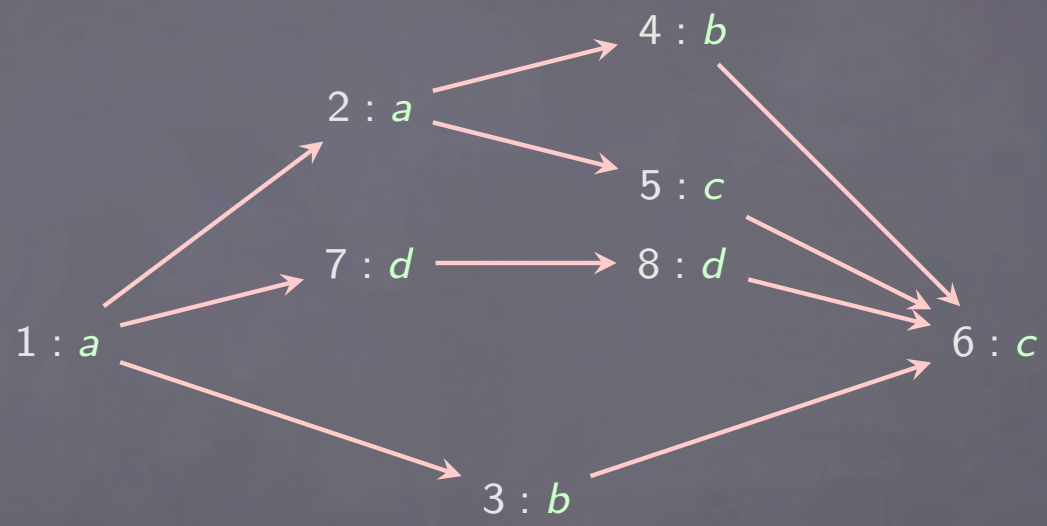






Pomsets with boxes



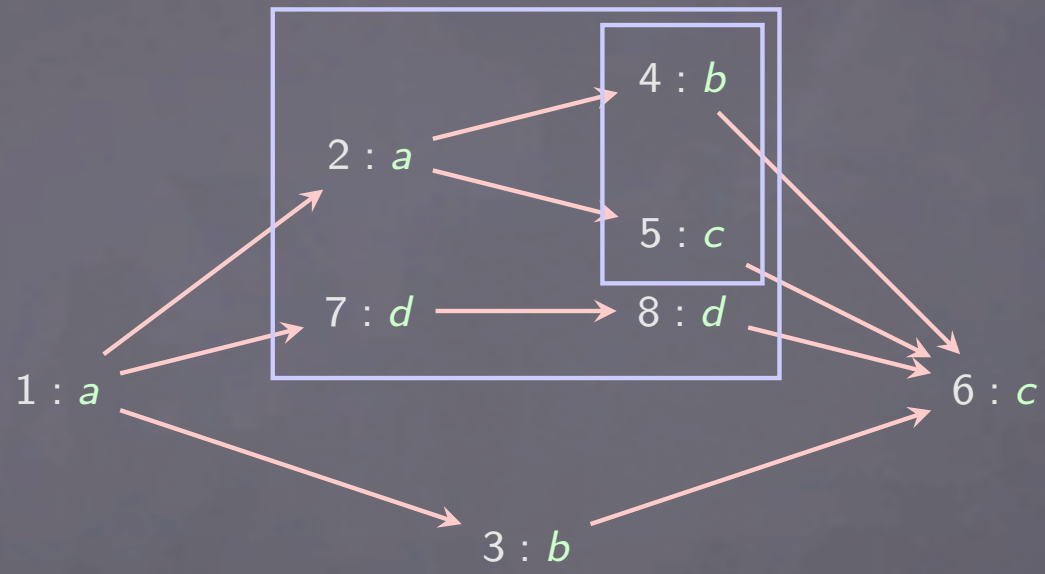
finite set of events

labeling function
$: E_P \to A$

$P = \langle E_P, \leqslant_P, \lambda_P \rangle$

partial order
$\subseteq E_P \times E_P$

Pomsets with boxes



finite set of events

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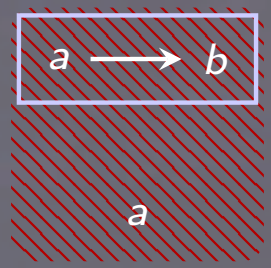
partial order $\subseteq E_P \times E_P$

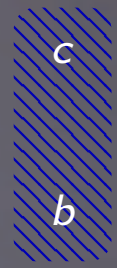
set of boxes $\subseteq \mathcal{P}^{\neq \emptyset}(E_P)$

Combining pomsets

$a =$ 

$1 =$ 

$P_1 =$ 

$P_2 =$ 

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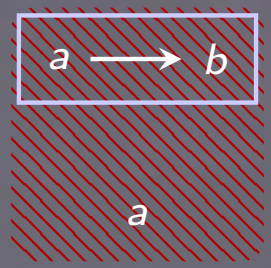
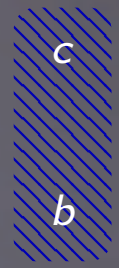
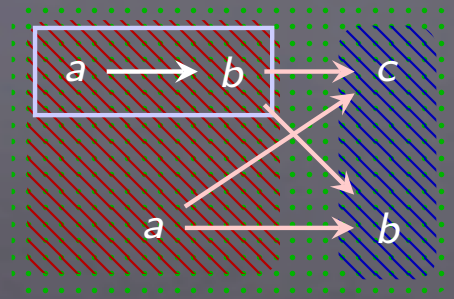
$P_1 =$ 

$P_2 =$ 

$P_1; P_2 =$ 

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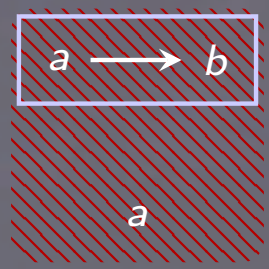
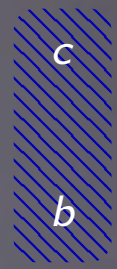
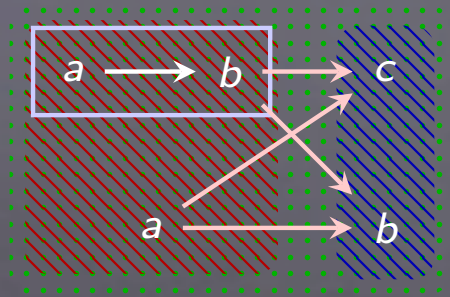
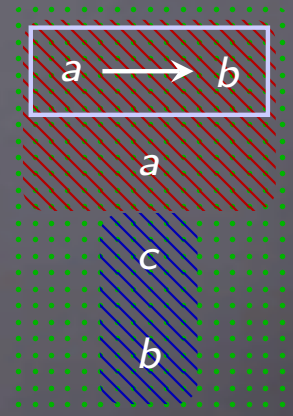
$P_1 =$ 

$P_2 =$ 

$P_1; P_2 =$ 

$P_1 \parallel P_2 =$ 

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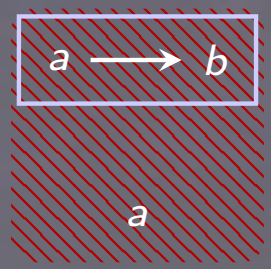
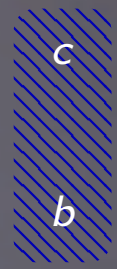
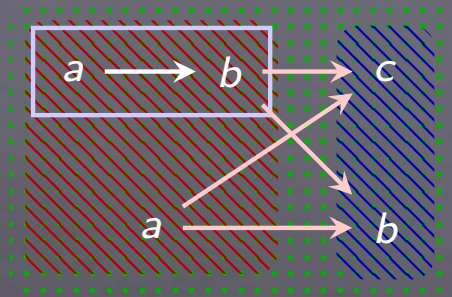
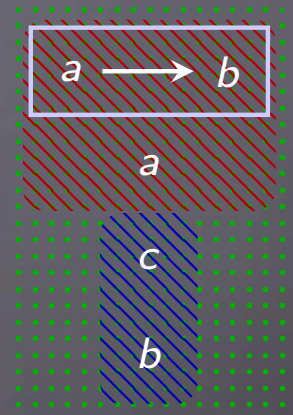
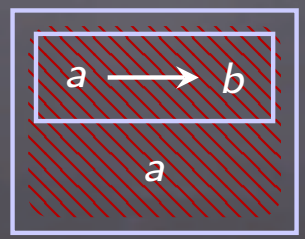
$P_1 =$ 

$P_2 =$ 

$P_1; P_2 =$ 

$P_1 \parallel P_2 =$ 

$[P_1] =$ 

Characterisation of SP-pomsets with boxes

Question

What pomsets can be built with the signature $\langle A, \cdot, \|, [-] \rangle$?

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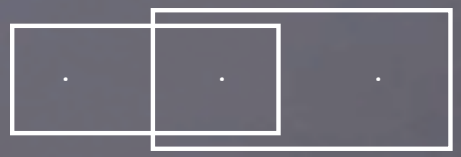
What pomsets can be built with the signature $\langle A, \cdot, \|, [-] \rangle$?

Those that do not include the following patterns:



Axiomatisation of isomorphism

$$BiMon_{\square} \vdash \quad P;(Q;R) = (P;Q);R$$
$$BiMon_{\square} \vdash \quad P;1 = 1;P$$

$$BiMon_{\square} \vdash \quad P \parallel (Q \parallel R) = (P \parallel Q) \parallel R$$
$$BiMon_{\square} \vdash \quad P \parallel Q = Q \parallel P$$
$$BiMon_{\square} \vdash \quad P \parallel 1 = 1 \parallel P$$

$$BiMon_{\square} \vdash \quad [1] = 1$$
$$BiMon_{\square} \vdash \quad [[P]] = [P]$$

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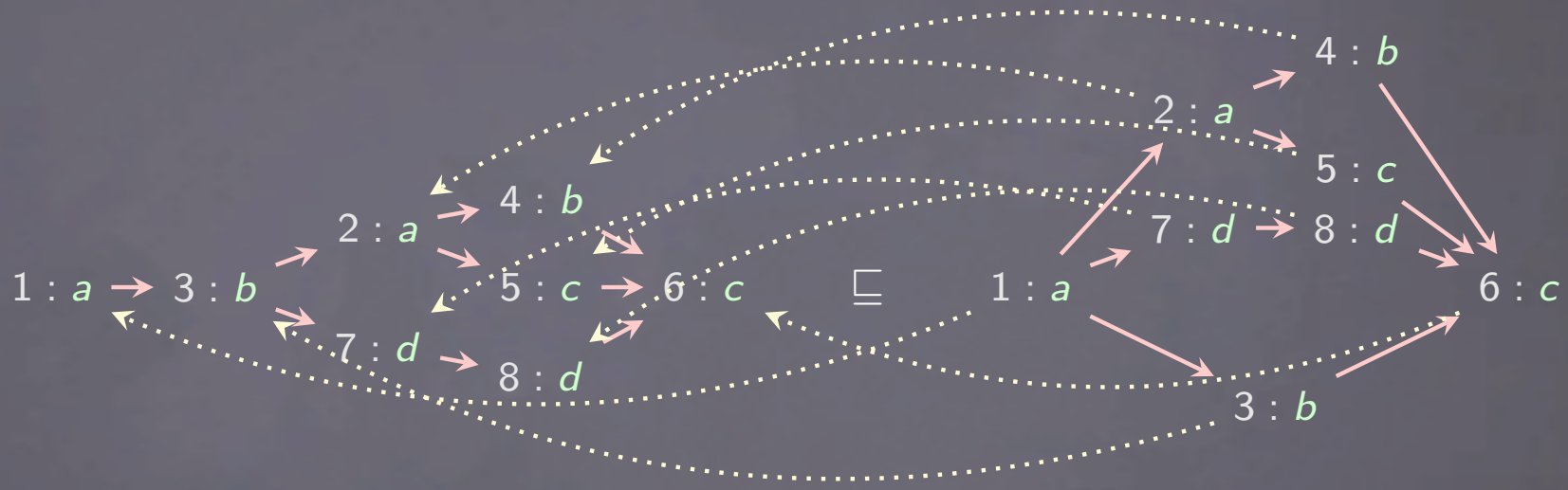
$$BiMon_{\square} \vdash \quad P \parallel 1 = 1 \parallel P$$

$$BiMon_{\square} \vdash \quad [1] = 1$$
$$BiMon_{\square} \vdash \quad [[P]] = [P]$$

Theorem

$$P \equiv Q \Leftrightarrow \ BiMon_{\square} \vdash P = Q.$$

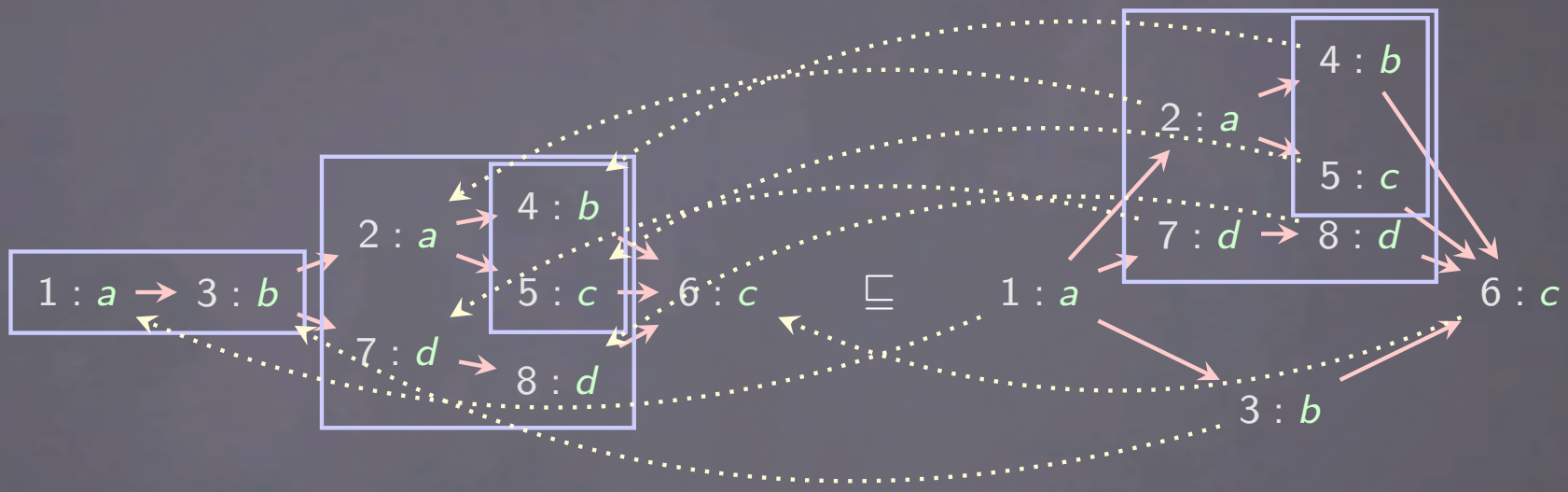
Subsumption with boxes



$P \sqsubseteq Q$ when there is a homomorphism from Q to P, i.e. a bijective map $\varphi : E_Q \to E_P$ such that

1. $\lambda_P \circ \varphi = \lambda_Q$
2. $\varphi(\leq_Q) \subseteq \leq_P$

Subsumption with boxes



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2) $\varphi(\leq_Q) \subseteq \leq_P$
3) $\varphi(\mathcal{B}_P) \subseteq \mathcal{B}_Q$

Axiomatisation of subsumption

$$BiMon_{\square} \subseteq CMon_{\square}$$

$$CMon_{\square} \vdash \quad (P \parallel Q);(R \parallel S) \sqsubseteq (P;R) \parallel (Q;S)$$

$$CMon_{\square} \vdash \quad [P] \sqsubseteq P$$

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$$CMon_{\square} \vdash \quad [P] \sqsubseteq P$$

Theorem

$$P \sqsubseteq Q \Leftrightarrow \ CMon_{\square} \vdash P \sqsubseteq Q.$$

Example: voting protocol

$$
\begin{aligned}
\texttt{VoteProc} &\coloneqq \texttt{Choose};\texttt{Publish} \\
\texttt{Choose} &\coloneqq [\texttt{Vote}(1)] \parallel \cdots \parallel [\texttt{Vote}(n)] \\
\texttt{Vote}(i) &\coloneqq \sum_{1 \leqslant j \leqslant k} \boxtimes_{i,j}; \hookrightarrow_j; \boxplus; \boxdot_j \\
\texttt{Publish} &\coloneqq \text{☎}_1 \parallel \cdots \parallel \text{☎}_n
\end{aligned}
$$

v_i : voter i
c_j : counter for candidate j
☎$_i$: the contents of counters $c_1, \ldots, c_k$ is sent to voter v_i
$\boxtimes_{i,j}$: voter v_i chooses counter c_j
$\hookrightarrow_j$: the content of counter c_j is loaded into a local variable
$\boxplus$: the local variable is incremented
$\boxdot_j$: the content of the local variable is stored in counter c_j

Non-deterministic programs

$$e, f ::= 0 \mid 1 \mid a \mid e \cdot f \mid e \parallel f \mid e + f \mid [e]\,.$$

Axioms

- Bisemiring with boxes: $SR_{\Box} = BiMon_{\Box} +$

$$e + f = f + e \qquad e + (f + g) = (e + f) + g \qquad e + 0 = e$$

$$e \cdot (f + g) = e \cdot f + e \cdot g \qquad (e + f) \cdot g = e \cdot g + f \cdot g$$

$$e \parallel (f + g) = e \parallel f + e \parallel g \qquad (e + f) \parallel g = e \parallel g + f \parallel g$$

$$e \cdot 0 = 0 \cdot e = 0 \qquad e \parallel 0 = 0$$

$$[e + f] = [e] + [f] \qquad [0] = 0$$

- Concurrent semiring with boxes:
  $CSR_{\Box} = SR_{\Box}$ + interchange + $[e] \leq e$.

Completeness theorems

$[\![e]\!]$: finite set of pomsets with boxes.

$[\![e]\!] \subseteq [\![f]\!]$: $\forall P \in [\![e]\!], \exists Q \in [\![f]\!] : P \equiv Q$.

$[\![e]\!] \sqsubseteq [\![f]\!]$: $\forall P \in [\![e]\!], \exists Q \in [\![f]\!] : P \sqsubseteq Q$.

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Theorem

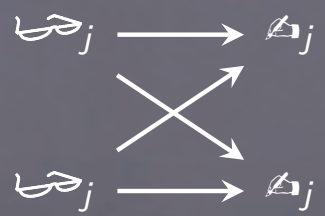
$$[\![e]\!] \subseteq [\![f]\!] \Leftrightarrow SR_{\Box} \vdash e + f = f.$$

$$[\![e]\!] \sqsubseteq [\![f]\!] \Leftrightarrow CSR_{\Box} \vdash e + f = f.$$

Mutual exclusion (II)

$$
\begin{aligned}
\texttt{VoteProc} &:= \texttt{Choose}; \texttt{Publish} \\
\texttt{Choose} &:= [\texttt{Vote}(1)] \parallel \cdots \parallel [\texttt{Vote}(n)] \\
\texttt{Vote}(i) &:= \textstyle\sum_{1 \leqslant j \leqslant k} \boxtimes_{i,j}; \hookrightarrow_j; \boxplus; \lightning_j \\
\texttt{Publish} &:= \text{☎}_1 \parallel \cdots \parallel \text{☎}_n
\end{aligned}
$$

Breaking mutual exclusion
$\Leftrightarrow$ admitting an execution with the following "pattern":



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\texttt{Vote}\,(i) &:= \textstyle\sum_{1\leqslant j\leqslant k} \boxtimes_{i,j}; 👓_j; ♔; ✍_j\\
\texttt{Publish} &:= ☎_1 \parallel \cdots \parallel ☎_n
\end{aligned}$$

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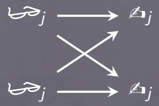
$\Leftrightarrow$ admitting an execution with the following "pattern":



Problem: cannot be expressed as *program* $\sqsubseteq$ *property*.

Pomset logic

$$\varphi, \psi ::= \bot \mid a \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \varphi \blacktriangleright \psi \mid \varphi \star \psi \mid [\varphi] \mid (\!|\varphi|\!)$$

- $P \models_{\sqsupseteq} \varphi \blacktriangleright \psi$ iff $\exists P_1, P_2$ s.t. $P \sqsupseteq P_1 \cdot P_2$, $P_1 \models_{\sqsupseteq} \varphi$ and $P_2 \models_{\sqsupseteq} \psi$
- $P \models_{\sqsupseteq} \varphi \star \psi$ iff $\exists P_1, P_2$ s.t. $P \sqsupseteq P_1 \parallel P_2$, $P_1 \models_{\sqsupseteq} \varphi$ and $P_2 \models_{\sqsupseteq} \psi$
- $P \models_{\sqsupseteq} [\varphi]$ iff $\exists Q$ s.t. $P \sqsupseteq [Q]$ and $Q \models_{\sqsupseteq} \varphi$
- $P \models_{\sqsupseteq} (\!|\varphi|\!)$ iff $\exists P', Q$ s.t. $P \sqsupseteq P'$ and $P' \Rsh Q$ and $Q \models_{\sqsupseteq} \varphi$.

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Theorem

$$P \sqsupseteq Q \Leftrightarrow \forall \varphi, \, (P \models_{\sqsupseteq} \varphi \Rightarrow Q \models_{\sqsupseteq} \varphi)\,.$$

$$P \equiv Q \Leftrightarrow \forall \varphi, \, (P \models_{\equiv} \varphi \Leftrightarrow Q \models_{\equiv} \varphi)\,.$$

Pomset logic

Remark

When the satisfaction relation is $\models_{\equiv}$ we may use negations, with:

- $P \models_{\equiv} \neg\varphi$ iff $P \not\models_{\equiv} \varphi$.

$$\varphi, \psi ::= \bot \mid a \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \varphi \blacktriangleright \psi \mid \varphi \star \psi \mid [\varphi] \mid (\!|\varphi|\!)$$

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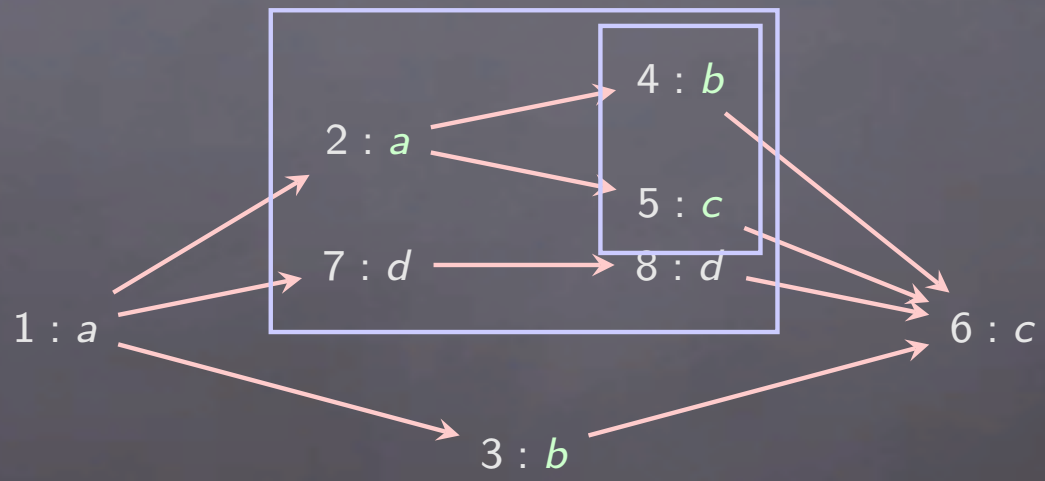
Contexts

☞ $P \models_{\sqsupseteq} (\!|\varphi|\!)$ iff $\exists P', Q$ such that $P \sqsupseteq P'$ and $P' \trianglerighteq Q$ and $Q \models_{\sqsupseteq} \varphi$.

Subpomset

$P \trianglerighteq Q$ iff there exists a set of events $A \subseteq E_P$ s.t. $Q \equiv P|_A$.

$A = \{2, 3, 4, 5\}$



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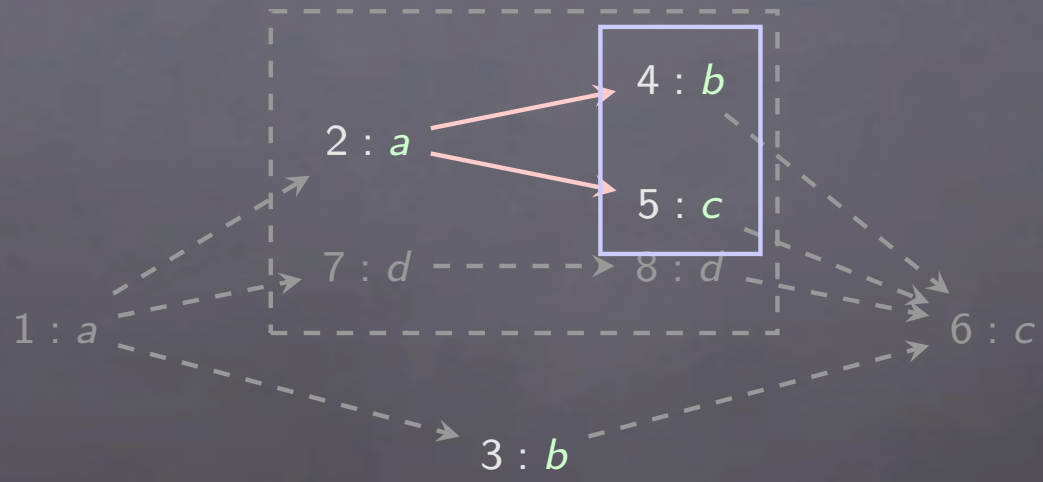

Extension to non-deterministic programs

Definition

$$e \models^{\forall}_{R} \varphi \text{ iff } \forall P \in [\![e]\!], P \models_{R} \varphi$$

$$e \models^{\exists}_{R} \varphi \text{ iff } \exists P \in [\![e]\!], P \models_{R} \varphi$$

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Theorem

$$\begin{aligned} SR_{\square} \vdash e \leq f &\Leftrightarrow \forall \varphi,\ e \models^{\exists}_{\equiv} \varphi \Rightarrow f \models^{\exists}_{\equiv} \varphi \\ &\Leftrightarrow \forall \varphi,\ f \models^{\forall}_{\equiv} \varphi \Rightarrow e \models^{\forall}_{\equiv} \varphi \\ CSR_{\square} \vdash e \leq f &\Leftrightarrow \forall \varphi,\ e \models^{\exists}_{\sqsupseteq} \varphi \Rightarrow f \models^{\exists}_{\sqsupseteq} \varphi \\ &\Leftrightarrow \forall \varphi,\ f \models^{\forall}_{\sqsubseteq} \varphi \Rightarrow e \models^{\forall}_{\sqsubseteq} \varphi \end{aligned}$$

Mutual exclusion (III)

$$\begin{aligned}\texttt{VoteProc} &:= \texttt{Choose}; \texttt{Publish}\\ \texttt{Choose} &:= [\texttt{Vote}(1)] \parallel \cdots \parallel [\texttt{Vote}(n)]\\ \texttt{Vote}(i) &:= \textstyle\sum_{1 \leqslant j \leqslant k} \boxtimes_{i,j}; \hookrightarrow_j; \boxminus; \lightning_j\\ \texttt{Publish} &:= \text{☎}_1 \parallel \cdots \parallel \text{☎}_n\end{aligned}$$

Breaking mutual exclusion
$\Leftrightarrow$ admitting an execution with the following "pattern":

$$\mathsf{conflict}_j := \langle\!\langle (\hookrightarrow_j \star \hookrightarrow_j) \blacktriangleright (\lightning_j \star \lightning_j) \rangle\!\rangle.$$

$$\begin{aligned}\texttt{VoteProc}' &:= \texttt{Choose}'; \texttt{Publish}\\ \texttt{Choose}' &:= \texttt{Vote}(1) \parallel \cdots \parallel \texttt{Vote}(n)\end{aligned}$$

$$\texttt{VoteProc} \not\models^{\exists}_{\sqsupseteq} \mathsf{conflict}_j \qquad \texttt{VoteProc}' \models^{\exists}_{\sqsupseteq} \mathsf{conflict}_j.$$

Spacio-Temporal separation

$$\begin{aligned}\texttt{VoteProc} &:= \texttt{Choose}; \texttt{Publish}\\ \texttt{Choose} &:= [\texttt{Vote}(1)] \parallel \cdots \parallel [\texttt{Vote}(n)]\\ \texttt{Vote}(i) &:= \textstyle\sum_{1 \leqslant j \leqslant k} \boxtimes_{i,j}; \hookrightarrow_j; \lightning; \lightning_j\\ \texttt{Publish} &:= \text{☎}_1 \parallel \cdots \parallel \text{☎}_n\end{aligned}$$

$$\mathsf{SendAfterVote} := \neg\left(\left\|\bigvee_i \text{☎}_i\right\|\right) \blacktriangleright \neg\left(\left\|\bigvee_{i,j} \boxtimes_{i,j}\right\|\right).$$

$$\texttt{Choose} \models^{\forall}_{\equiv} \neg\left(\left\|\bigvee_i \text{☎}_i\right\|\right) \text{ and } \texttt{Publish} \models^{\forall}_{\equiv} \neg\left(\left\|\bigvee_{i,j} \boxtimes_{i,j}\right\|\right)$$

$$\Rightarrow \texttt{VoteProc} \models^{\forall}_{\equiv} \mathsf{SendAfterVote}$$

$$\mathsf{VoteThenSend} := \left\|\left(\bigvee_j \boxtimes_{1,j} \blacktriangleright \text{☎}_1\right) \star \ldots \star \left(\bigvee_j \boxtimes_{n,j} \blacktriangleright \text{☎}_n\right)\right\|$$

$$\texttt{VoteProc} \models^{\forall}_{\equiv} \mathsf{VoteThenSend}.$$

Unique votes

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$$\texttt{VoteProc} \not\models \overset{\exists}{\sqsubseteq} \bigvee_{j',j} \langle\!|[\langle\!|\text{✍}_j \star \text{✍}_{j'}|\!\rangle]|\!\rangle$$

Further work

Algebra:

Logic:

Further work

Algebra:

- ☞ Where's my star?

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- Merge axiom:

$$[a; b] \leq [a] ; [b]$$

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- Logic of Behaviour
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  - Pomset logic relies on an abstract notion of "behaviour" instead.
  - What kind of properties can be expressed?

That's all folks!

Thank you!

See more at:
http://paul.brunet-zamansky.fr