Pomsets with boxes

towards Atomic Concurrent Klenee Algebra

Highlights of logic, games and automata

September 2019

Paul Brunet, David Pym
University College London

UCL

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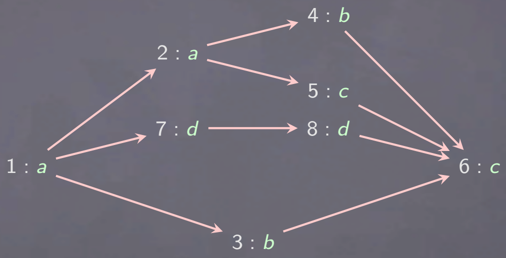
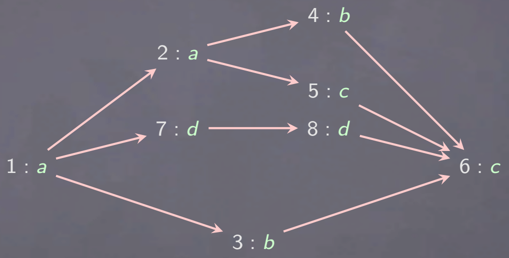
Pomsets



Pomsets

A is some alphabet of actions.



finite set of events

labeling function $: E_P \to A$

$$P = \langle E_P, \leqslant_P, \lambda_P \rangle$$

partial order $\subseteq E_P \times E_P$

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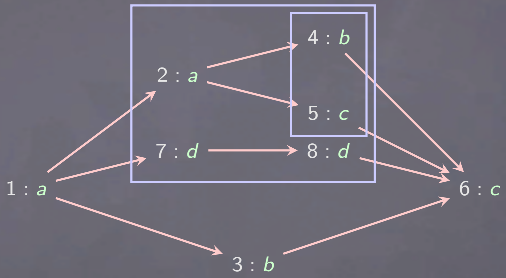

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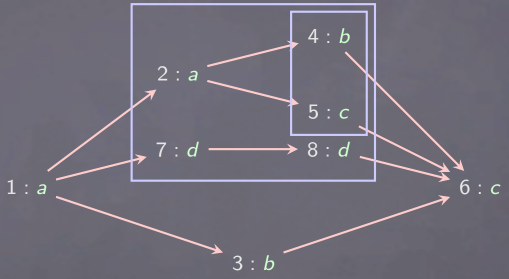

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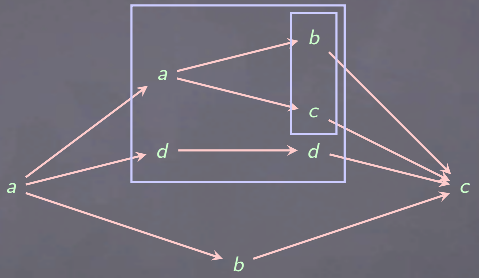

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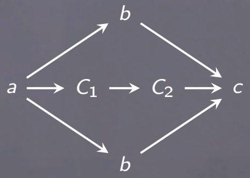

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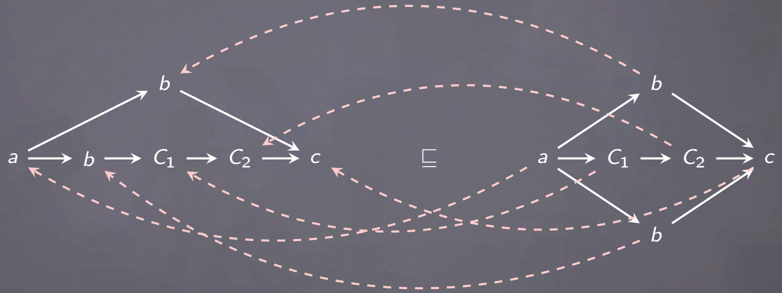

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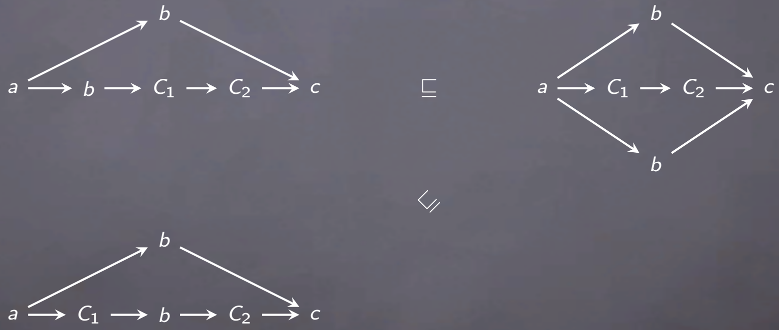
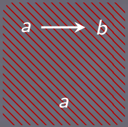


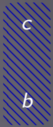
Pomset subsumption

Building Pomsets

$a =$ 

$1 =$ 

$P_1 =$ 

$P_2 =$ 

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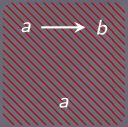
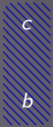
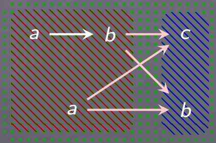
$P_1 =$ 

$P_2 =$ 

$P_1; P_2 =$ 

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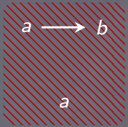
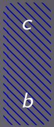
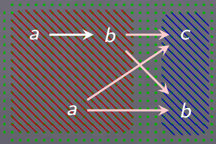
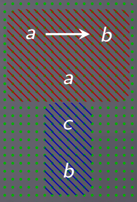
$P_1 =$ 

$P_2 =$ 

$P_1; P_2 =$ 

$P_1 \parallel P_2 =$ 

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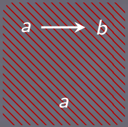
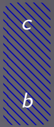
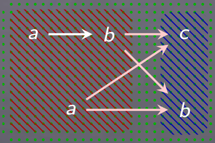
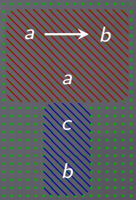
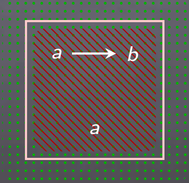
$P_1 =$ 

$P_2 =$ 

$P_1; P_2 =$ 

$P_1 \parallel P_2 =$ 

$[P_1] =$ 

Characterisation of SP-Pomsets

Question

What pomsets can be built with the signature $A, ;, \|, [-]$?

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What pomsets can be built with the signature $A, ;, \|, [-]$?

Those that do not include the following patterns:

Axiomatisation of isomorphism

$$BSP \vdash \quad P;(Q;R) = (P;Q);R$$
$$BSP \vdash \quad P;1 = 1;P$$

$$BSP \vdash \quad P \parallel (Q \parallel R) = (P \parallel Q) \parallel R$$
$$BSP \vdash \quad P \parallel Q = Q \parallel P$$
$$BSP \vdash \quad P \parallel 1 = 1 \parallel P$$

$$BSP \vdash \quad [1] = 1$$
$$BSP \vdash \quad [[P]] = [P]$$

Theorem

$$P \equiv Q \Leftrightarrow \; BSP \vdash P = Q.$$

Axiomatisation of subsumption

$$BSP_{\sqsubseteq} \vdash \quad (P \parallel Q);(R \parallel S) \sqsubseteq (P;R) \parallel (Q;S)$$

$$BSP_{\sqsubseteq} \vdash \quad [P] \sqsubseteq P$$

Theorem

$$P \sqsubseteq Q \Leftrightarrow BSP_{\sqsubseteq} \vdash P \sqsubseteq Q.$$

That's all folks!

Thank you!

See more at:
http://paul.brunet-zamansky.fr