Recent developments

in

concurrent Kleene algebra

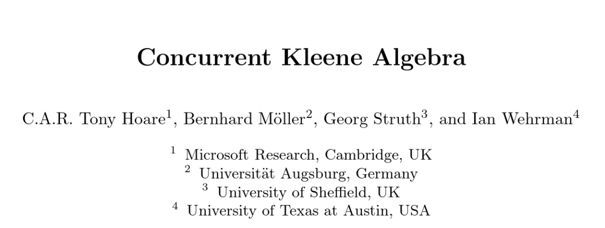
Séminaire PPS - Paris

Juin 2020

Paul Brunet
University College London

UCL

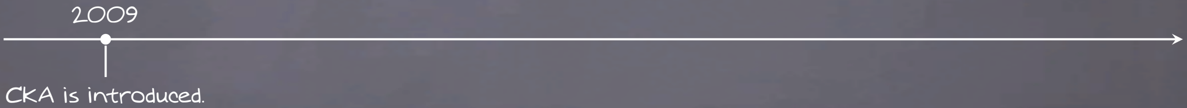
Concurrent Kleene Algebra



Concurrent Kleene Algebra

C.A.R. Tony Hoare[1], Bernhard Möller[2], Georg Struth[3], and Ian Wehrman[4]

[1] Microsoft Research, Cambridge, UK
[2] Universität Augsburg, Germany
[3] University of Sheffield, UK
[4] University of Texas at Austin, USA



2009

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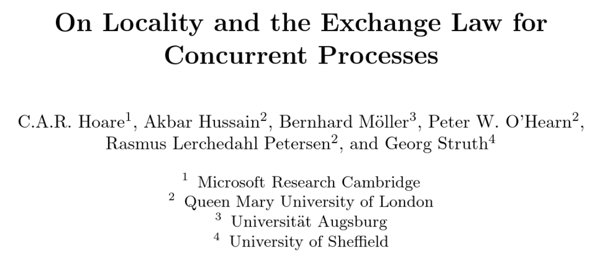
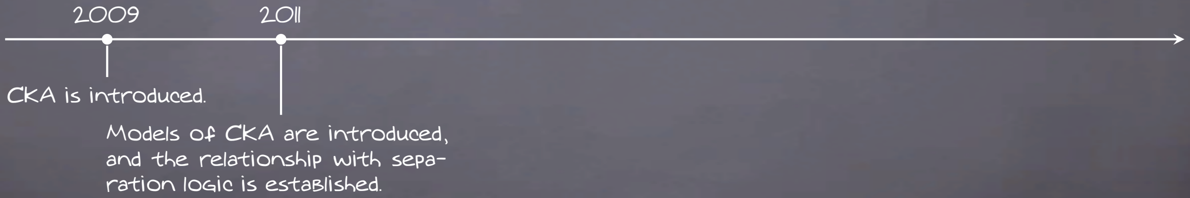
Concurrent Kleene Algebra



On Locality and the Exchange Law for Concurrent Processes

C.A.R. Hoare[1], Akbar Hussain[2], Bernhard Möller[3], Peter W. O'Hearn[2], Rasmus Lerchedahl Petersen[2], and Georg Struth[4]

[1] Microsoft Research Cambridge
[2] Queen Mary University of London
[3] Universität Augsburg
[4] University of Sheffield



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Models of CKA are introduced, and the relationship with separation logic is established.

Concurrent Kleene Algebra

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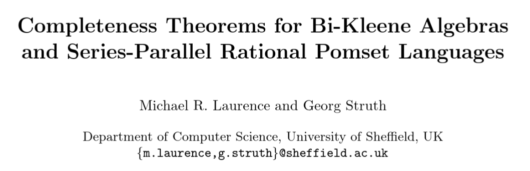
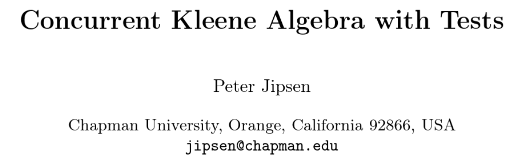


Completeness Theorems for Bi-Kleene Algebras and Series-Parallel Rational Pomset Languages

Michael R. Laurence and Georg Struth

Department of Computer Science, University of Sheffield, UK
{m.laurence,g.struth}@sheffield.ac.uk

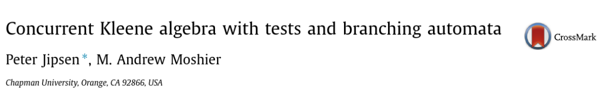


Concurrent Kleene Algebra with Tests

Peter Jipsen

Chapman University, Orange, California 92866, USA
jipsen@chapman.edu

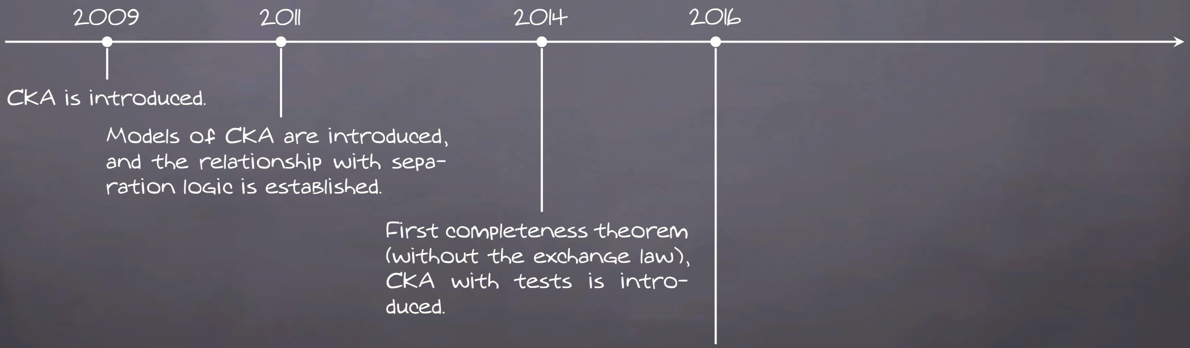
Concurrent Kleene Algebra



Concurrent Kleene algebra with tests and branching automata

Peter Jipsen*, M. Andrew Moshier

Chapman University, Orange, CA 92866, USA



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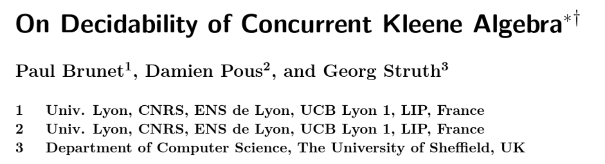
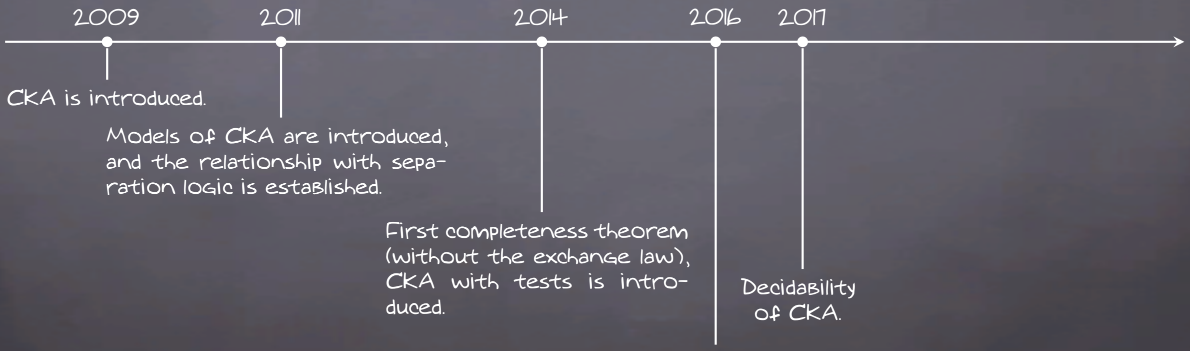
Concurrent Kleene Algebra



On Decidability of Concurrent Kleene Algebra*†

Paul Brunet[1], Damien Pous[2], and Georg Struth[3]

1 Univ. Lyon, CNRS, ENS de Lyon, UCB Lyon 1, LIP, France
2 Univ. Lyon, CNRS, ENS de Lyon, UCB Lyon 1, LIP, France
3 Department of Computer Science, The University of Sheffield, UK



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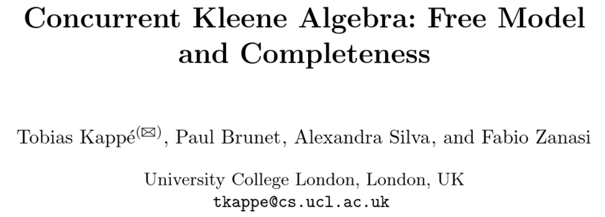
Concurrent Kleene Algebra



Concurrent Kleene Algebra: Free Model and Completeness

Tobias Kappé(✉), Paul Brunet, Alexandra Silva, and Fabio Zanasi

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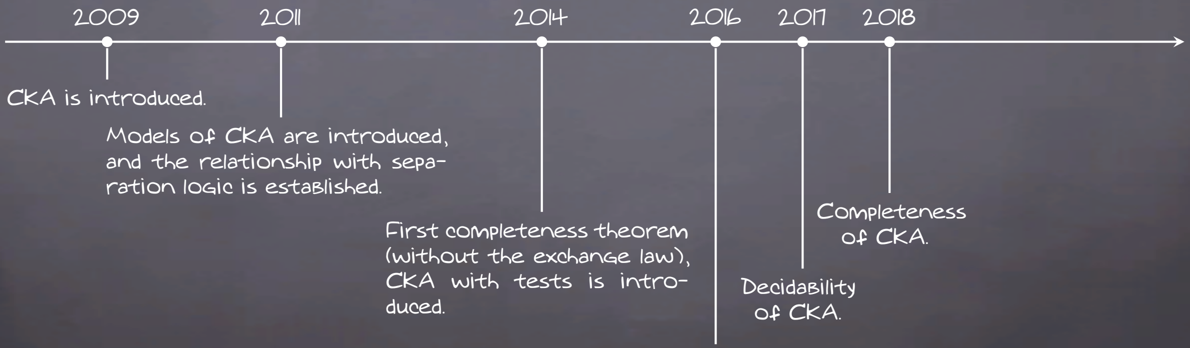

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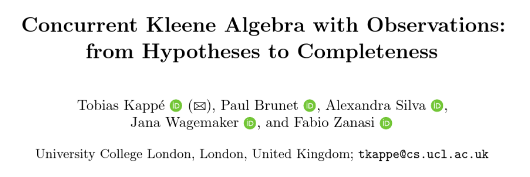
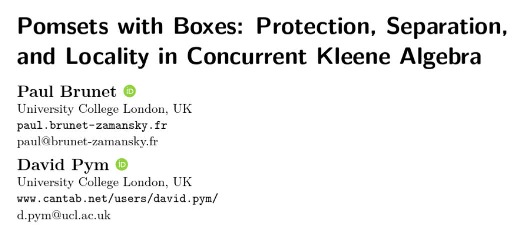
Concurrent Kleene Algebra



Concurrent Kleene Algebra with Observations: from Hypotheses to Completeness

Tobias Kappé (✉), Paul Brunet, Alexandra Silva, Jana Wagemaker, and Fabio Zanasi

University College London, London, United Kingdom; tkappe@cs.ucl.ac.uk



Pomsets with Boxes: Protection, Separation, and Locality in Concurrent Kleene Algebra

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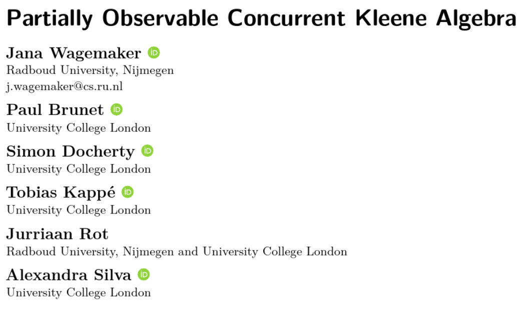
David Pym
University College London, UK
www.cantab.net/users/david.pym/
d.pym@ucl.ac.uk



Partially Observable Concurrent Kleene Algebra

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Simon Docherty
University College London

Tobias Kappé
University College London

Jurriaan Rot
Radboud University, Nijmegen and University College London

Alexandra Silva
University College London

2

2016

2017

2018

2020

CKA is

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intro-

Decidability of CKA.

Completeness of CKA.

CKA with observations, partially observable CKA, CKA with boxes.

Second paper on CKAT, correcting some mistakes from the first one.

Kleene algebra: the algebra of regular expressions

$$e, f \in E_A ::= 0 \mid 1 \mid a \mid e \cdot f \mid e + f \mid e^\star$$

$$[\![\,\cdot\,]\!] : E_A \to \mathcal{P}\left(A^\star\right)$$

The axioms of KA

$$\begin{array}{ccc}
e + e = e & e + f = f + e & e + (f + g) = (e + f) + g \\
e + 0 = 0 & e \cdot 1 = e = 1 \cdot e & e \cdot (f \cdot g) = (e \cdot f) \cdot g \\
e \cdot 0 = 0 = 0 \cdot e & e \cdot (f + g) = e \cdot f + e \cdot g & (e + f) \cdot g = e \cdot g + f \cdot g
\end{array}$$

$$e^\star = 1 + e \cdot e^\star \qquad e \cdot f \leq f \Rightarrow e^\star \cdot f \leq f$$

Theorem

$$KA \vdash e = f \Leftrightarrow [\![e]\!] = [\![f]\!].$$

Kozen, "A completeness theorem for Kleene algebras and the algebra of regular events", LiCS '90

KAT: the algebra of imperative programs

$$e, f \in E_{A\cup B_T} ::= 0 \;\mid\; 1 \;\mid\; a \in A \;\mid\; t \in B_T \;\mid\; e \cdot f \;\mid\; e + f \;\mid\; e^\star$$

$$t, t_1, t_2 \in B_T ::= \top \;\mid\; \bot \;\mid\; \alpha \in T \;\mid\; t_1 \wedge t_2 \;\mid\; t_1 \vee t_2 \;\mid\; \neg t$$

The axioms of KAT

- The axioms of KA.
- For tests, the axioms of boolean algebra.
- The following "glue" axioms:

$$t_1 \vee t_2 = t_1 + t_2 \qquad t_1 \wedge t_2 = t_1 \cdot t_2 \qquad \top = 1 \qquad \bot = 0$$

Kozen & Smith, "Kleene algebra with tests: Completeness and decidability", CSL '96

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abort execution

$$t, t_1, t_2 \in B_T ::= \top \mid \bot \mid \alpha \in T \mid t_1 \wedge t_2 \mid t_1 \vee t_2 \mid \neg t$$

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abort execution   atomic action   atomic test

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KAT: the algebra of imperative programs

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sequential composition

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abort execution　　atomic action　　atomic test　　non-deterministic choice

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KAT: the algebra of imperative programs

non-deterministic loop

skip

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KAT

☞ Free algebra: languages over Guarded Strings, i.e. $2^T \cdot (A \cdot 2^T)^\star$.

$$\begin{tikzcd}
\begin{array}{|c|c|}\hline \alpha_1 & 1 \\ \alpha_2 & 0 \\ \alpha_3 & 1 \\ \alpha_4 & 1 \\ \hline\end{array} \arrow[r, "a"] & \begin{array}{|c|c|}\hline \alpha_1 & 1 \\ \alpha_2 & 0 \\ \alpha_3 & 0 \\ \alpha_4 & 1 \\ \hline\end{array} \arrow[r, "b"] & \begin{array}{|c|c|}\hline \alpha_1 & 0 \\ \alpha_2 & 0 \\ \alpha_3 & 1 \\ \alpha_4 & 0 \\ \hline\end{array} \arrow[r, "1"] & \begin{array}{|c|c|}\hline \alpha_1 & 1 \\ \alpha_2 & 1 \\ \alpha_3 & 1 \\ \alpha_4 & 0 \\ \hline\end{array}
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- Encodes a simple `While` language:

$$\texttt{if } b \texttt{ then } p \texttt{ else } q \mapsto b \cdot p + \neg b \cdot q \qquad\qquad \texttt{while } b \texttt{ do } p \mapsto (b \cdot p)^\star \cdot \neg b$$

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Can we do the same for concurrent programs?

Outline

I. Concurrent Kleene Algebra

II. CKA with observations

III. Partially observable CKA

IV. CKA with boxes

V. Conclusions

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☞ I. Concurrent Kleene Algebra

II. CKA with observations

III. Partially observable CKA

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bi-Kleene Algebra

$$e, f ::= 1 \mid 0 \mid x \mid e \cdot f \mid e \parallel f \mid e + f \mid e^{\star} \mid e^{!}$$

Definition

A bi-Kleene algebra is a structure $\langle A, 0, 1, \cdot, \parallel, +, \star, ! \rangle$ such that:

- ☞ $\langle A, 0, 1, \cdot, +, \star \rangle$ is a KA
- ☞ $\langle A, 0, 1, \parallel, +, ! \rangle$ is a commutative KA.

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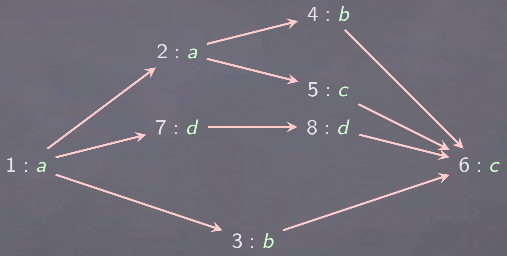
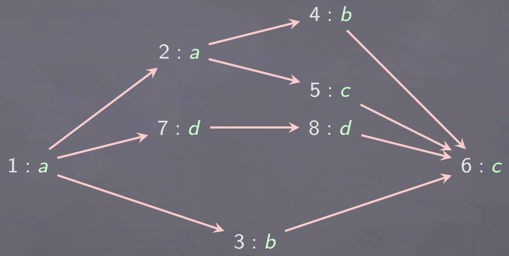
What is the free bi-KA?

Pomsets: concurrent traces



Pomsets: concurrent traces

A is some alphabet of actions.



finite set of events

labeling function
$: E_P \to A$

$$P = \langle E_P, \leqslant_P, \lambda_P \rangle$$

partial order
$\subseteq E_P \times E_P$

Pomsets: concurrent traces

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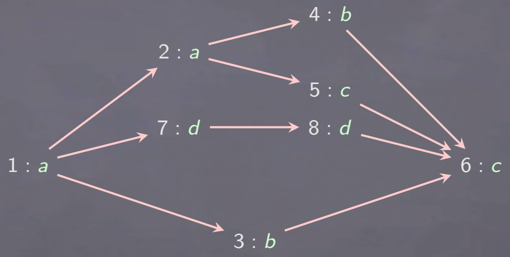

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$\boxed{\text{Up-to isomorphism } \equiv.}$

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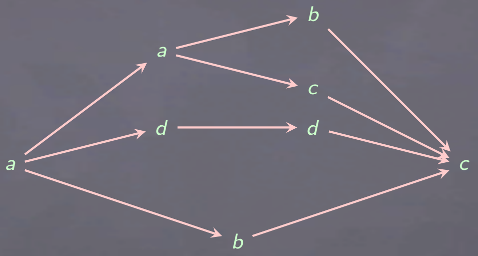

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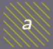
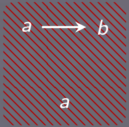
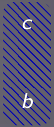
Up-to isomorphism $\equiv$.

Combining pomsets

$a =$


$1 =$

$P_1 =$


$P_2 =$


Combining pomsets

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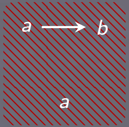
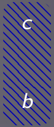
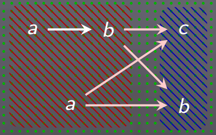
$P_1 =$ 

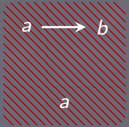
$P_2 =$ 

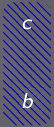
$P_1; P_2 =$ 

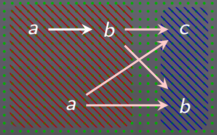
Combining pomsets

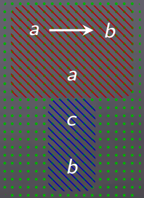
$a =$

$1 =$

$P_1 =$


$P_2 =$


$P_1 ; P_2 =$


$P_1 \parallel P_2 =$


Completeness of biKA

$$
\begin{array}{rlrl}
\llbracket 1 \rrbracket & \coloneqq \{1\} & \llbracket 0 \rrbracket & \coloneqq \emptyset \\
\llbracket x \rrbracket & \coloneqq \{x\} & \llbracket e + f \rrbracket & \coloneqq \llbracket e \rrbracket \cup \llbracket f \rrbracket \\
\llbracket e \cdot f \rrbracket & \coloneqq \{ P ; Q \mid P \in \llbracket e \rrbracket,\ Q \in \llbracket f \rrbracket \} & \llbracket e \parallel f \rrbracket & \coloneqq \{ P \parallel Q \mid P \in \llbracket e \rrbracket,\ Q \in \llbracket f \rrbracket \} \\
\llbracket e^{\star} \rrbracket & \coloneqq \{ P_1 ; \cdots ; P_n \mid n \in \mathbb{N},\ P_i \in \llbracket e \rrbracket \} & \llbracket e^{!} \rrbracket & \coloneqq \{ P_1 \parallel \cdots \parallel P_n \mid n \in \mathbb{N},\ P_i \in \llbracket e \rrbracket \}
\end{array}
$$

Theorem

$$biKA \vdash e = f \Leftrightarrow \llbracket e \rrbracket \equiv \llbracket f \rrbracket.$$

Laurence & Struth, "Completeness Theorems for Bi-Kleene Algebras and Series-Parallel Rational Pomset Languages", RAMiCS '14

Concurrent Kleene Algebra

Interchange law

$$(a \parallel b) \cdot (c \parallel d) \leq (a \cdot c) \parallel (b \cdot d).$$

No parallel iteration

CKA

A concurrent Kleene algebra is a weak bi-Kleene algebra $\langle A, 0, 1, \cdot, \parallel, +, \star \rangle$ satisfying the interchange law.

Interleavings and subsumption

Interchange law

$$(a \parallel b) \cdot (c \parallel d) \leq (a \cdot c) \parallel (b \cdot d).$$

$$\begin{tikzcd}
a \arrow[r] \arrow[dr] & c & & a \arrow[r] & c \\
& & \sqsubseteq & & \\
b \arrow[r] \arrow[ur] & d & & b \arrow[r] & d
\end{tikzcd}$$

$P \sqsubseteq Q$ when there is a homomorphism from Q to P, i.e. a bijective map $\varphi : E_Q \to E_P$ such that $\lambda_P \circ \varphi = \lambda_Q$ and $\varphi(\leq_Q) \subseteq \leq_P$.

$$L^{\sqsubseteq} := \{P \mid \exists Q \in L : P \sqsubseteq Q\}.$$

Completeness and decidability of CKA

Theorem

The problem of testing whether two given expressions e, f denote the same closed language is ExpSpace-complete.

B., Pous, & Struth, "On Decidability of Concurrent Kleene Algebra", CONCUR '17

Theorem

$$CKA \vdash e = f \Leftrightarrow [\![e]\!]^{\sqsubseteq} = [\![f]\!]^{\sqsubseteq}.$$

Kappé, B., Silva, & Zanasi, "Concurrent Kleene Algebra: Free Model and Completeness", ESOP '18

Outline

I. Concurrent Kleene Algebra

II. CKA with observations

III. Partially observable CKA

IV. CKA with boxes

V. Conclusions

CKAT

Slogan

A KAT is a KA with a boolean sub-algebra.
A CKAT is a CKA with a boolean sub-algebra.

CKAT

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$$t \cdot p \cdot \neg t$$

CKAT

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$$t \cdot p \cdot \neg t = (1 \parallel t) \cdot (p \parallel 1) \cdot \neg t$$

CKAT

Slogan

A KAT is a KA with a boolean sub-algebra.
A CKAT is a CKA with a boolean sub-algebra.

$$\begin{aligned} t \cdot p \cdot \neg t &= (1 \parallel t) \cdot (p \parallel 1) \cdot \neg t \\ &\leq ((1 \cdot p) \parallel (t \cdot 1)) \cdot \neg t \end{aligned}$$

CKAT

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$$\begin{aligned} t \cdot p \cdot \neg t &= (1 \parallel t) \cdot (p \parallel 1) \cdot \neg t \\ &\leq ((1 \cdot p) \parallel (t \cdot 1)) \cdot \neg t \\ &= (p \parallel t) \cdot (1 \parallel \neg t) \end{aligned}$$

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t \cdot p \cdot \neg t &= (1 \parallel t) \cdot (p \parallel 1) \cdot \neg t \\
&\leq ((1 \cdot p) \parallel (t \cdot 1)) \cdot \neg t \\
&= (p \parallel t) \cdot (1 \parallel \neg t) \\
&\leq (p \cdot 1) \parallel (t \cdot \neg t) \\
&= p \parallel (t \wedge \neg t)
\end{aligned}$$

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CKAT: DOOMED!

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&\leq (p \cdot 1) \parallel (t \cdot \neg t) \\
&= p \parallel (t \wedge \neg t) \\
&= p \parallel \bot = p \parallel 0 = 0
\end{aligned}$$

↔ For every program and every assertion, the triple $\{t\}\, p\, \{t\}$ holds.
↔ Every test is invariant under every program.

Who's to blame?

$$\begin{aligned} t \cdot p \cdot \neg t &\leq p \parallel (t \cdot \neg t) && \text{(CKA axioms)} \\ &= p \parallel (t \wedge \neg t) && (\wedge = \cdot) \\ &= p \parallel \bot && \text{(Boolean axioms)} \\ &= p \parallel 0 = 0 && (\bot = 0 + \text{CKA axioms}) \end{aligned}$$

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$$a \wedge b \;=\; a \cdot b$$

"If we observe a, and then observe b without any action in between, then both observations are made on the same state. Therefore that state simultaneously satisfies a and b."

Who's to blame?

$$
\begin{aligned}
t \cdot p \cdot \neg t &\leq p \parallel (t \cdot \neg t) && \text{(CKA axioms)}\\
&= p \parallel (t \wedge \neg t) && \boxed{\cancel{(\wedge = \cdot)}}\\
&= p \parallel \bot && \text{(Boolean axioms)}\\
&= p \parallel 0 = 0 && (\bot = 0 + \text{CKA axioms})
\end{aligned}
$$

$$\cancel{a \wedge b \; = \; a \cdot b}$$

"If we observe a, and then observe b without any action in between, then both observations are made on the <u>same</u> state. Therefore that state simultaneously satisfies a and b."

$$a \wedge b \;\leq\; a \cdot b$$

CKAT

$$e, f \in E_{A\cup B_T} ::= 0 \mid 1 \mid a \in A \mid t \in B_T \mid e \cdot f \mid e \parallel f \mid e + f \mid e^\star$$

$$t, t_1, t_2 \in B_T ::= \top \mid \bot \mid \alpha \in T \mid t_1 \wedge t_2 \mid t_1 \vee t_2 \mid \neg t$$

The axioms of CKAT

- ☞ The axioms of CKA.
- ☞ For tests, the axioms of boolean algebra.
- ☞ The following "glue" axioms:

$$t_1 \vee t_2 = t_1 + t_2 \qquad t_1 \wedge t_2 = t_1 \cdot t_2 \qquad \top = 1 \qquad \bot = 0$$

CKAO

$$e, f \in E_{A \cup B_T} ::= 0 \mid 1 \mid a \in A \mid t \in B_T \mid e \cdot f \mid e \parallel f \mid e + f \mid e^\star$$

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The axioms of CKAO

☞ The axioms of CKA.

☞ For tests, the axioms of boolean algebra.

☞ The following "glue" axioms:

$$t_1 \vee t_2 = t_1 + t_2 \qquad t_1 \wedge t_2 \leq t_1 \cdot t_2 \qquad \bot = 0$$

Interlude: (C)KA with hypotheses

☞ E_A: Expressions over A.

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CKA+H

$$\frac{biKA \vdash e = f}{CKA + H \vdash e = f} \qquad \frac{e \leq f \in H}{CKA + H \vdash e \leq f}$$

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CKA+H

$$\frac{biKA \vdash e = f}{CKA + H \vdash e = f} \qquad \frac{e \leq f \in H}{CKA + H \vdash e \leq f}$$

H-closure

$$L \subseteq L{\downarrow}^H \qquad \frac{e \leq f \in H \qquad C[\![f]\!] \subseteq L{\downarrow}^H}{C[\![e]\!] \subseteq L{\downarrow}^H}$$

Interlude: (C)KA with hypotheses

Theorem

$$CKA + H \vdash e = f \Rightarrow [\![e]\!]{\downarrow}^{H} = [\![f]\!]{\downarrow}^{H}$$

Doumane, Kuperberg, Pous, & Pradic, "Kleene Algebra with Hypotheses", FoSSaCS '19

Kappé, B., Silva, Wagemaker, & Zanasi, "Concurrent Kleene Algebra with Observations: from Hypotheses to Completeness", FoSSaCS '20

Completeness of CKAO

CKAO as an instance of CKA+H

- $exch = \{(e \parallel f) \cdot (g \parallel h) \leq (e \cdot g) \parallel (f \cdot h) \ \mid e, f, g, h \in E_{A \cup B_T}\}$;
- $bool = \{p \leq q \ \mid Bool \vdash p \leq q\}$;
- $contr = \{p \wedge q \leq p \cdot q \ \mid p, q \in B_T\}$;
- $glue = \{\bot \leq 0\} \cup \{p \vee q \leq p + q \ \mid p, q \in B_T\}$;
- $obs = exch \cup bool \cup contr \cup glue.$

$$CKAO \vdash e = f \Leftrightarrow CKA + obs \vdash e = f$$

By the previous (generic) theorem, we get $CKAO \vdash e = f \Rightarrow [\![e]\!]{\downarrow}^{obs} = [\![f]\!]{\downarrow}^{obs}$.

Theorem

$$CKAO \vdash e = f \Leftrightarrow [\![e]\!]{\downarrow}^{obs} = [\![f]\!]{\downarrow}^{obs}.$$

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☞ III. Partially observable CKA

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Litmus test: sequential consistency

```
    { r0 == 0 && r1 == 0 }

      x := 1   ║  y := 1
      r0 := y  ║  r1 := x

{ !( r0 == 1 || r1 == 1 ) }
```

Ingredients:

- ☞ Assignments $x \leftarrow 1$
- ☞ Observations $r_0 = 0$

Algebra of observations

What boolean algebra can we get out of observations of the shape $r_0 = 0$?

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Answer: sets of memory states $\text{Var} \to \text{Val}$.

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Answer:

$$\sum_{s \in State} s \cdot (v \leftarrow n) \cdot s[v \mapsto n].$$

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Answer:

$$\sum_{s \in State} s \cdot (v \leftarrow n) \cdot s[v \mapsto n].$$

Problem: how do we execute those in parallel?

$$\begin{array}{|c|c|} x & 0 \\ y & 0 \end{array} \xrightarrow{x \leftarrow 1} \begin{array}{|c|c|} x & 1 \\ y & 0 \end{array}$$

$$\begin{array}{|c|c|} x & 0 \\ y & 0 \end{array} \xrightarrow{y \leftarrow 1} \begin{array}{|c|c|} x & 0 \\ y & 1 \end{array}$$

Algebra of observations

Solution: Move to partial functions $\textsc{Var} \rightharpoonup \textsc{Val}$.

Algebra of observations

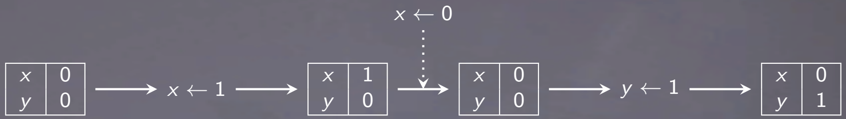
Solution: Move to partial functions $\textsc{Var} \rightharpoonup \textsc{Val}$.

Algebraically: Boolean algebra → Pseudo-complemented distributive lattice.

Same axioms as BA regarding $\vee, \wedge, \top, \bot$, plus:

$$p \leq \overline{q} \Leftrightarrow p \wedge q = \bot.$$

Causality vs compositionality



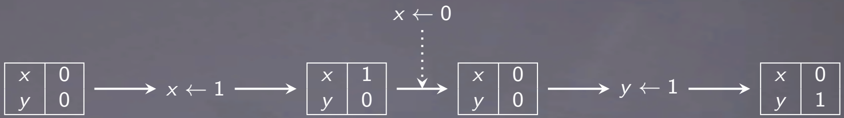
Causality vs compositionality

$$\begin{tikzcd}
 & & x \leftarrow 0 \arrow[d, dotted] & & \\
\begin{array}{|c|c|}\hline x & 0 \\ y & 0 \\\hline\end{array} \arrow[r, "x \leftarrow 1" description] & \begin{array}{|c|c|}\hline x & 1 \\ y & 0 \\\hline\end{array} \arrow[r, no head] & {} \arrow[r] & \begin{array}{|c|c|}\hline x & 0 \\ y & 0 \\\hline\end{array} \arrow[r, "y \leftarrow 1" description] & \begin{array}{|c|c|}\hline x & 0 \\ y & 1 \\\hline\end{array}
\end{tikzcd}$$

Solution: we need to explicitly close the system.

$$[\![e]\!] \quad \to \quad [\![e]\!] \cap \mathit{CausalPomsets}.$$

Causality vs compositionality



$x \leftarrow 0$

x	0
y	0

$\longrightarrow x \leftarrow 1 \longrightarrow$

x	1
y	0

$\longrightarrow$

x	0
y	0

$\longrightarrow y \leftarrow 1 \longrightarrow$

x	0
y	1

Solution: we need to explicitly close the system.

$$[\![e]\!] \quad \rightarrow \quad [\![e]\!] \cap \mathit{CausalPomsets}.$$

Litmus test:

$$t := (r_0 = 0 \wedge r_1 = 0) \cdot ((x \leftarrow 1 \cdot r_0 \leftarrow y) \parallel (y \leftarrow 1 \cdot r_1 \leftarrow x)) \cdot \overline{(r_0 = 1 \vee r_1 \vee 1)}$$

$$[\![t]\!] \cap \mathit{CausalPomsets} = \emptyset$$

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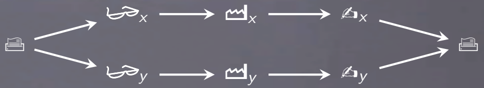
Mutual exclusion

```
        print(counter);
                ||
x:=counter;     ||     y:=counter;
x:=x+1;         ||     y:=y+1;
counter:=x;     ||     counter:=y;
                ||
        print(counter);
```



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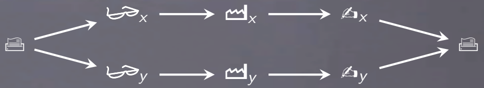

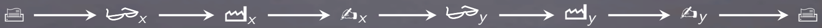

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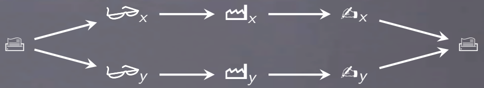

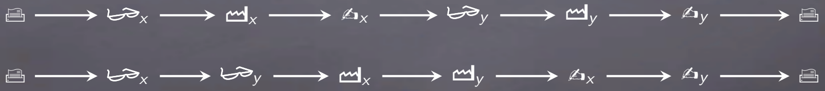

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Mutual exclusion

```
      print(counter);
atomic{          || atomic{
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  counter:=x;    ||   counter:=y;
}                || }
      print(counter);
```

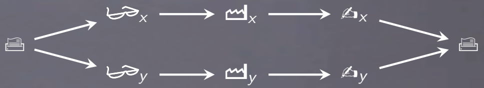

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🖶 ⟶ 👓$_x$ ⟶ 👓$_y$ ⟶ 🏭$_x$ ⟶ 🏭$_y$ ⟶ ✍$_x$ ⟶ ✍$_y$ ⟶ 🖶

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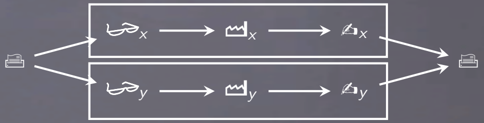

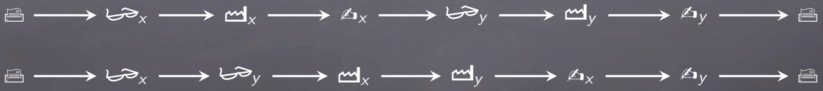

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🖶 → 👓x → 👓y → 🏭x → 🏭y → ✍x → ✍y → 🖶

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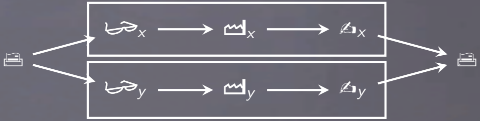
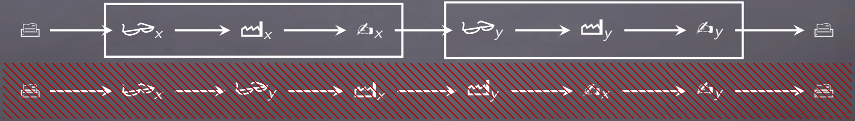
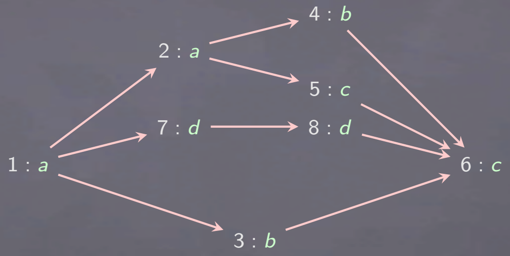




Pomsets with boxes



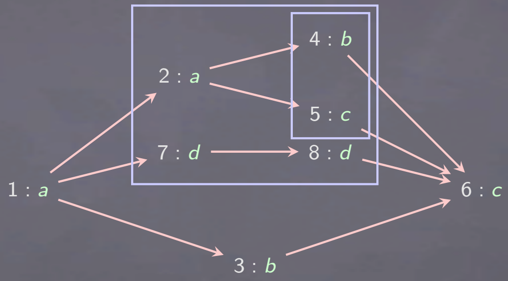
finite set of events

labeling function
$: E_P \to A$

$$P = \langle E_P, \leqslant_P, \lambda_P \quad \rangle$$

partial order
$\subseteq E_P \times E_P$

Pomsets with boxes



finite set of events

labeling function
$: E_P \to A$

$$P = \langle E_P, \leqslant_P, \lambda_P, B_P \rangle$$

partial order
$\subseteq E_P \times E_P$

set of boxes
$\subseteq \mathcal{P}^{\neq\emptyset}(E_P)$

Characterisation of SP-pomsets with boxes

Question

What pomsets can be built with the signature $\langle A, \cdot, \|, [-] \rangle$?

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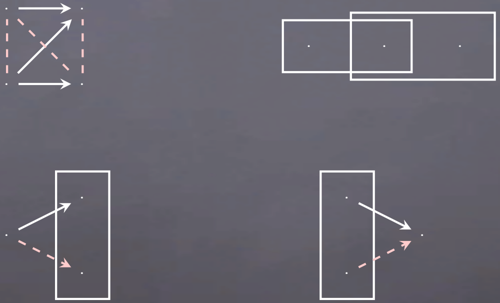
What pomsets can be built with the signature $\langle A, \cdot, \|, [-] \rangle$?

Those that do not include the following patterns:



Proved and developped in Coq

Axiomatisation of isomorphism

$$\begin{aligned}
BSP &\vdash & P; (Q; R) &= (P; Q); R \\
BSP &\vdash & P; 1 &= 1; P \\
\\
BSP &\vdash & P \parallel (Q \parallel R) &= (P \parallel Q) \parallel R \\
BSP &\vdash & P \parallel Q &= Q \parallel P \\
BSP &\vdash & P \parallel 1 &= 1 \parallel P \\
\\
BSP &\vdash & [1] &= 1 \\
BSP &\vdash & [[P]] &= [P]
\end{aligned}$$

Theorem

$$P \equiv Q \Leftrightarrow \; BSP \vdash P = Q.$$

Axiomatisation of isomorphism

Proved and developped in Coq

$$
\begin{aligned}
BSP &\vdash P; (Q; R) = (P; Q); R \\
BSP &\vdash P; 1 = 1; P \\
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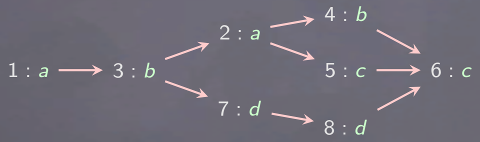
BSP &\vdash P \parallel (Q \parallel R) = (P \parallel Q) \parallel R \\
BSP &\vdash P \parallel Q = Q \parallel P \\
BSP &\vdash P \parallel 1 = 1 \parallel P \\
\\
BSP &\vdash [1] = 1 \\
BSP &\vdash [[P]] = [P]
\end{aligned}
$$

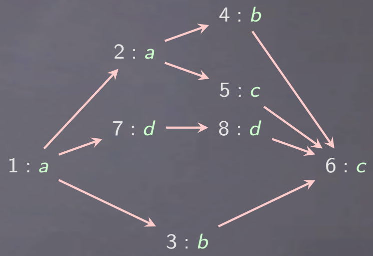
Theorem

$$P \equiv Q \Leftrightarrow \; BSP \vdash P = Q.$$

Subsumption with boxes



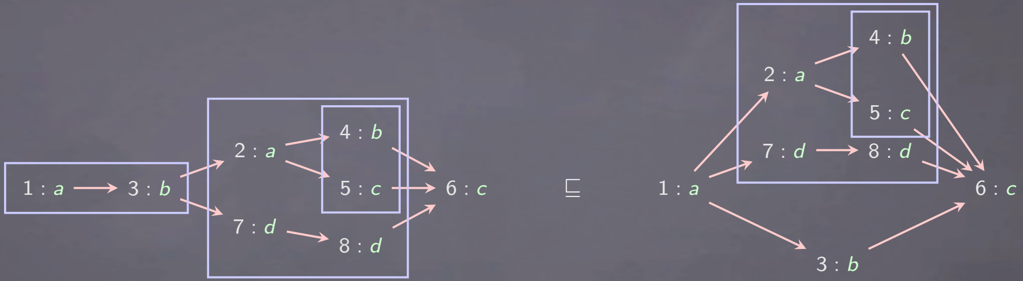
$\sqsubseteq$



$P \sqsubseteq Q$ when there is a homomorphism from Q to P, i.e. a bijective map $\varphi : E_Q \to E_P$ such that

1) $\lambda_P \circ \varphi = \lambda_Q$
2) $\varphi(\leq_Q) \subseteq \leq_P$

Subsumption with boxes



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1) $\lambda_P \circ \varphi = \lambda_Q$
2) $\varphi(\leq_Q) \subseteq \leq_P$
3) $\varphi(\mathcal{B}_P) \subseteq \mathcal{B}_Q$

Axiomatisation of subsumption

$$BSP_{\sqsubseteq} \vdash \qquad (P \parallel Q);(R \parallel S) \sqsubseteq (P;R) \parallel (Q;S)$$

$$BSP_{\sqsubseteq} \vdash \qquad [P] \sqsubseteq P$$

Theorem

$$P \sqsubseteq Q \Leftrightarrow \; BSP_{\sqsubseteq} \vdash P \sqsubseteq Q.$$

Axiomatisation of subsumption

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$$BSP_{\sqsubseteq} \vdash \qquad (P \parallel Q);(R \parallel S) \sqsubseteq (P;R) \parallel (Q;S)$$

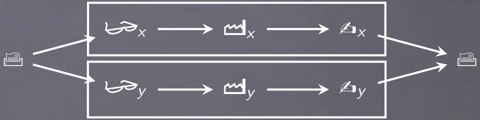
$$BSP_{\sqsubseteq} \vdash \qquad [P] \sqsubseteq P$$

Theorem

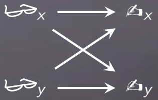
$$P \sqsubseteq Q \Leftrightarrow \; BSP_{\sqsubseteq} \vdash P \sqsubseteq Q.$$

Mutual exclusion (II)

```
        print(counter);
atomic{            ||  atomic{
   x:=counter;     ||     y:=counter;
   x:=x+1;         ||     y:=y+1;
   counter:=x;     ||     counter:=y;
}                  ||  }
        print(counter);
```



Breaking mutual exclusion ↔ admitting an execution with the following "pattern":



Pomset logic

$$\varphi, \psi ::= \bot \mid a \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \varphi \blacktriangleright \psi \mid \varphi \star \psi \mid [\varphi] \mid \llparenthesis \varphi \rrparenthesis$$

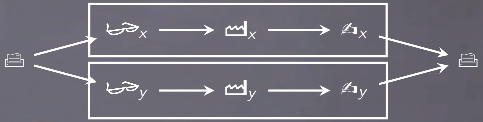
- ☞ $P \models \varphi \blacktriangleright \psi$ iff $\exists P_1, P_2$ such that $P \sqsupseteq P_1 \cdot P_2$ and $P_1 \models \varphi$ and $P_2 \models \psi$
- ☞ $P \models \varphi \star \psi$ iff $\exists P_1, P_2$ such that $P \sqsupseteq P_1 \parallel P_2$ and $P_1 \models \varphi$ and $P_2 \models \psi$
- ☞ $P \models [\varphi]$ iff $\exists Q$ such that $P \sqsupseteq [Q]$ and $Q \models \varphi$
- ☞ $P \models \llparenthesis \varphi \rrparenthesis$ iff $\exists P', Q$ such that $P \sqsupseteq P'$ and $P' \supsetplus Q$ and $Q \models \varphi$.

Theorem

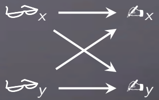
$$P \sqsupseteq Q \Leftrightarrow \forall \varphi, \left(P \models \varphi \Rightarrow Q \models \varphi\right).$$

Mutual exclusion (III)

```
        print(counter);
atomic{            || atomic{
  x:=counter;      ||   y:=counter;
  x:=x+1;          ||   y:=y+1;
  counter:=x;      ||   counter:=y;
}                  || }
        print(counter);
```



Breaking mutual exclusion ↔ admitting an execution with the following "pattern":



$\leftrightarrow P \models \llparenthesis(\text{👓}_x \star \text{👓}_y) \blacktriangleright (\text{✍}_x \star \text{✍}_y)\rrparenthesis$

Outline

I. Concurrent Kleene Algebra

II. CKA with observations

III. Partially observable CKA

IV. CKA with boxes

☞ V. Conclusions

Algebras with hypotheses

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Can we do better?

Logics of behaviour

☞ Traditional approaches to program logic rely on states
e.g. Hennessy-Milner Logic, (Propositional) Dynamic Logic...

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What does it mean?

Logics of behaviour

☞ Traditional approaches to program logic rely on states
e.g. Hennessy-Milner Logic, (Propositional) Dynamic Logic...

☞ Pomset logic relies on an abstract notion of "behaviour" instead.

What does it mean?

We have the hammer, where is the nail?

That's all folks!

Thank you!

See more at:
http://paul.brunet-zamansky.fr

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