On Decidability of Concurrent Kleene Algebra

CONCUR in Berlin – September 5-8, 2017

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University of Sheffield

Concurrent Kleene Algebra

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- **Kleene algebra**: rational expressions.
  - Can be used to reason about **sequential** programs.
  - Canonical model: regular languages, *i.e.* sets of words.

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- **Concurrent Kleene algebra**: series-rational expressions.
  - Can be used to reason about **concurrent** programs with a **refinement** order.
  - "Canonical" model: **downwards-closed** pomset languages.

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- **Kleene algebra**: rational expressions.
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Outline

I. Pomsets

II. Petri Nets

III. Summary and Outlook

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III. Summary and Outlook

Let's visit London!

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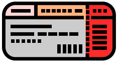

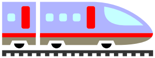

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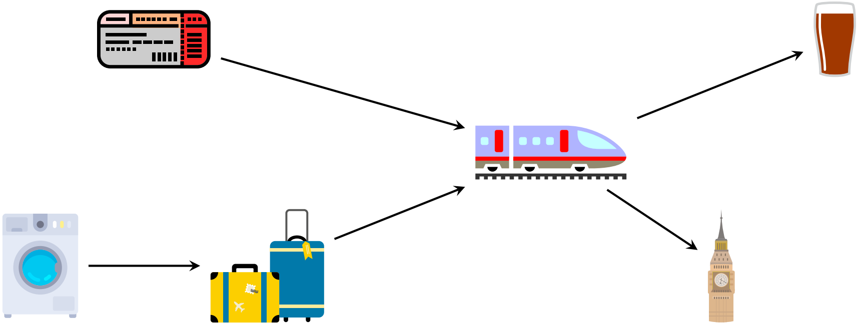

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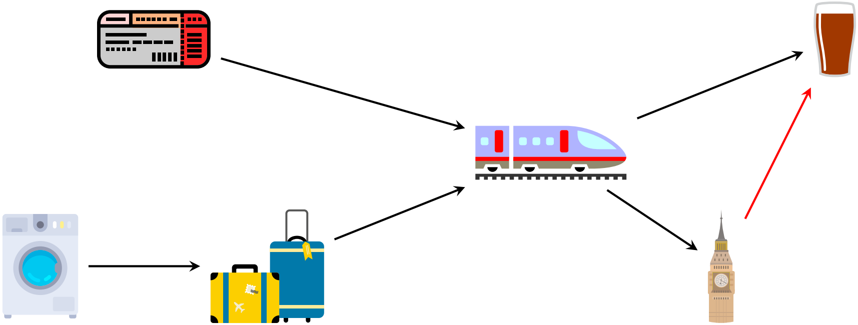


Pomsets products

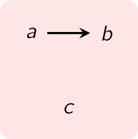
$$P_1 = \begin{array}{ccc} a & \longrightarrow & b \\ & c & \end{array} \qquad\qquad P_2 = \begin{array}{c} d \\ e \end{array}$$

Pomsets products

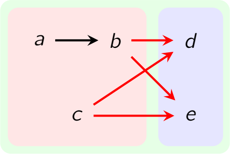
$$P_1 = \begin{tikzcd} a \arrow[r] & b \\ & c \end{tikzcd} \qquad P_2 = \begin{tikzcd} d \\ e \end{tikzcd}$$

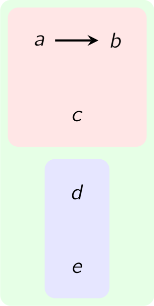
$$P_1 \cdot P_2 = \begin{tikzcd} a \arrow[r] & b \arrow[r] \arrow[dr] & d \\ & c \arrow[ur] \arrow[r] & e \end{tikzcd}$$

Pomsets products

$P_1 =$ 

$P_2 =$ 

$P_1 \cdot P_2 =$ 

$P_1 \parallel P_2 =$ 

Pomset order

$$
\begin{tikzcd}
a \arrow[r] \arrow[dr] & b \\
c \arrow[r] \arrow[ur] & d
\end{tikzcd}
\quad \sqsubseteq \quad
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Definition

$P_1 \sqsubseteq P_2$ if there is a function $\varphi : P_2 \to P_1$ such that:

1. φ is a bijection
2. φ preserves labels
3. φ preserves ordered pairs

Gischer, **The equational theory of pomsets**, *1988*

Grabowski, **On partial languages**, *1981*

Pomset order

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Notation

$^{\sqsubseteq}S := \{P \mid \exists P' \in S : P \sqsubseteq P'\}$.

Rational pomset languages

$$e, f \in \mathbb{E}_\Sigma ::= a \;\mid\; 0 \;\mid\; 1 \;\mid\; e \cdot f \;\mid\; e \parallel f \;\mid\; e + f \;\mid\; e^\star.$$

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$$\llbracket a \rrbracket := \left\{ \boxed{a} \right\}$$

$$\llbracket 0 \rrbracket := \emptyset$$

$$\llbracket e \cdot f \rrbracket := \llbracket e \rrbracket \cdot \llbracket f \rrbracket$$

$$\llbracket e^\star \rrbracket := \bigcup_{n \in \mathbb{N}} \llbracket e \rrbracket^n$$

$$\llbracket 1 \rrbracket := \left\{ \boxed{} \right\}$$

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Definition

A set of pomsets S is called a **rational pomset language** if there is an expression $e \in \mathbb{E}_\Sigma$ such that $S = \llbracket e \rrbracket$.

Two decision problems

biKA

Given two expressions e, f, are $[\![e]\!]$ and $[\![f]\!]$ equal?

CKA

Given two expressions e, f, are ${}^{\sqsubseteq}[\![e]\!]$ and ${}^{\sqsubseteq}[\![f]\!]$ equal?

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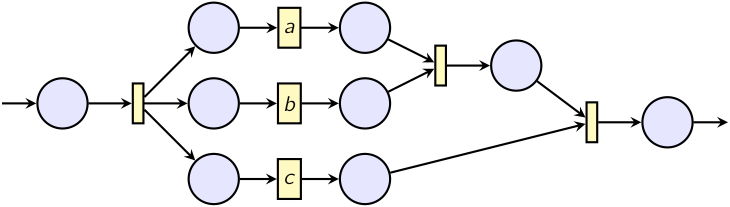
Labelled Petri nets



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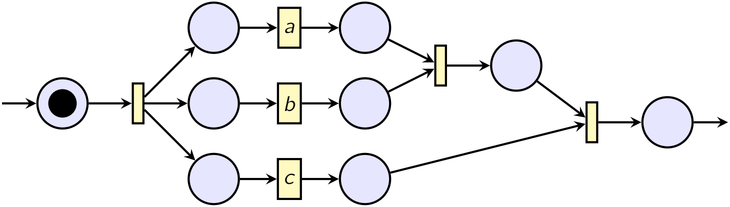

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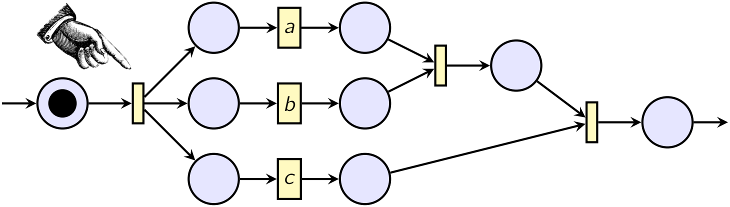

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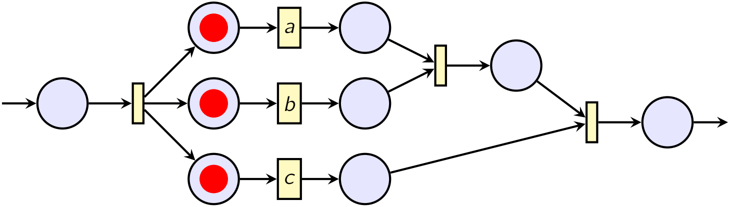

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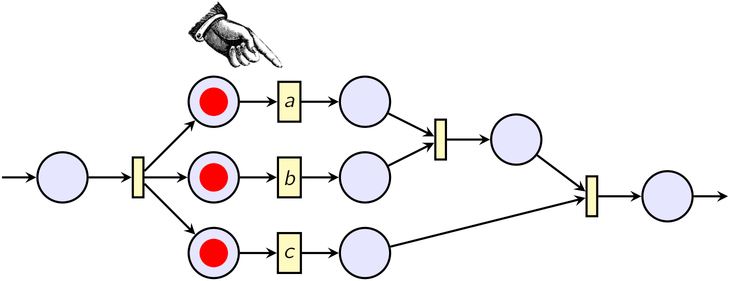

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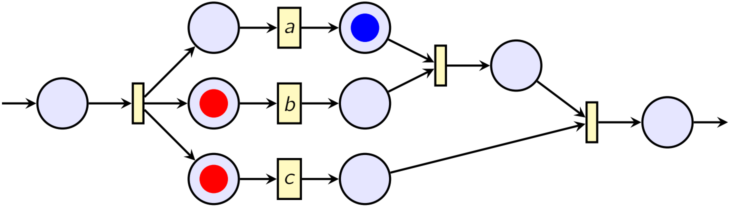

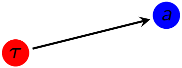

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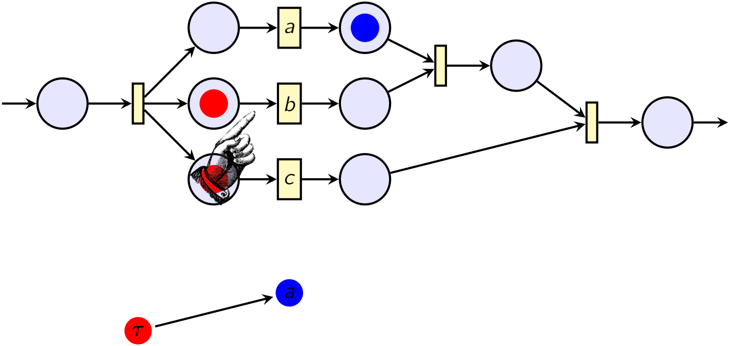

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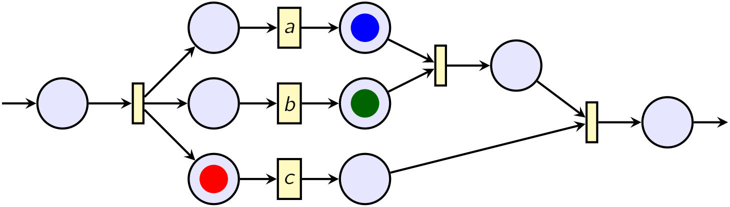

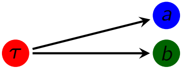

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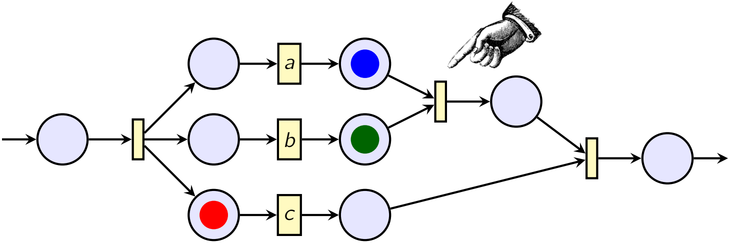

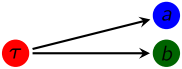

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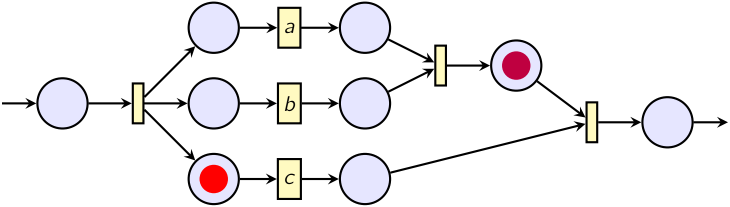

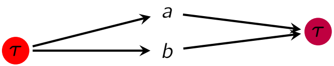

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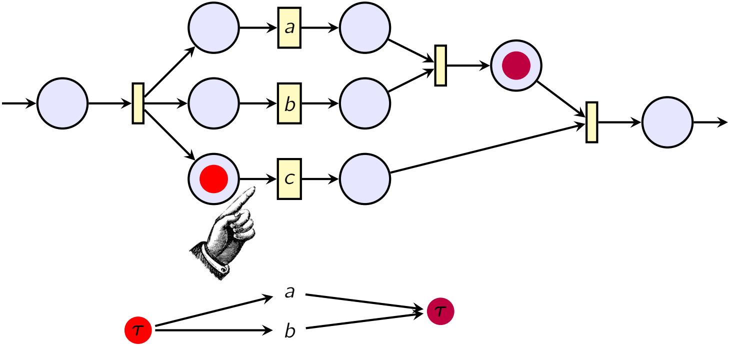

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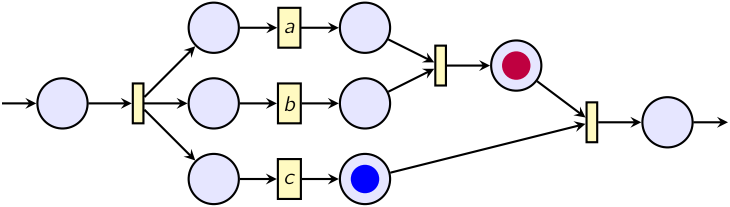

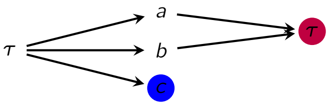

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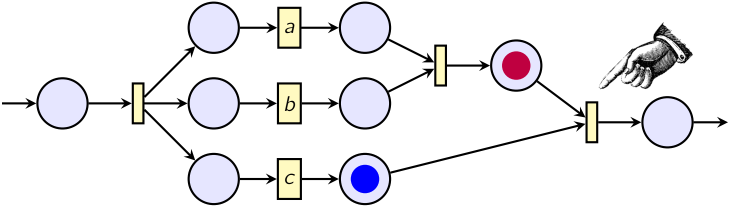

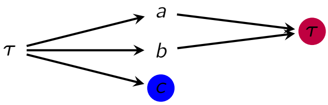

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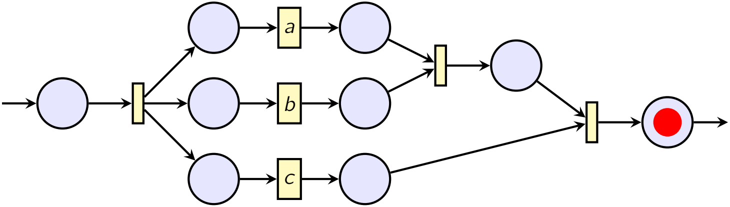

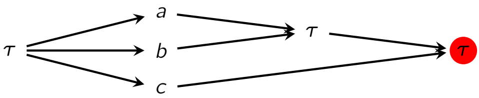

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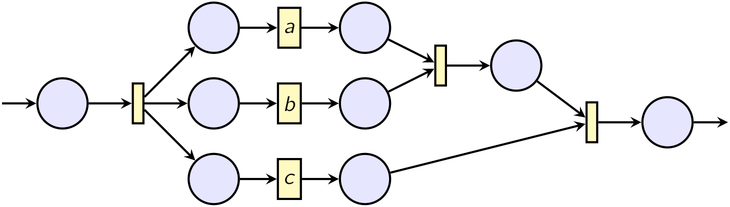

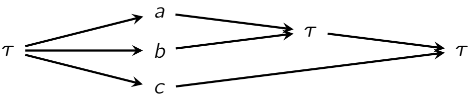

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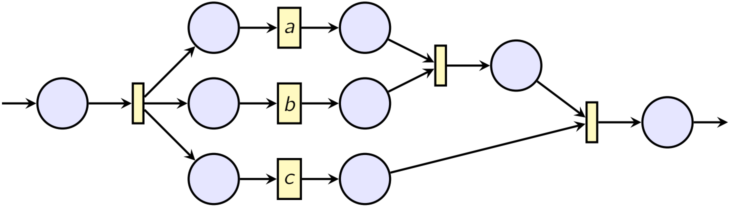
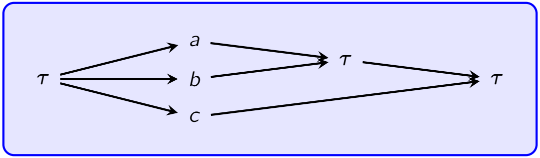
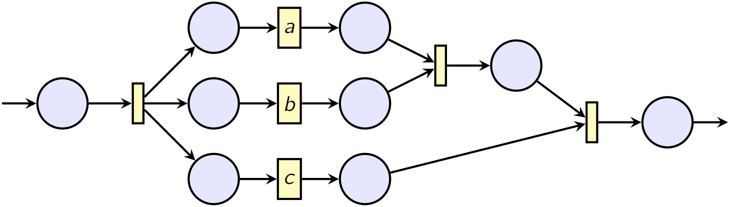
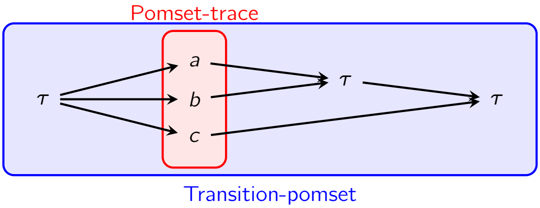




Transition-pomset

Labelled Petri nets





Recognisable pomset languages

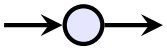
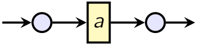
Language generated by a net

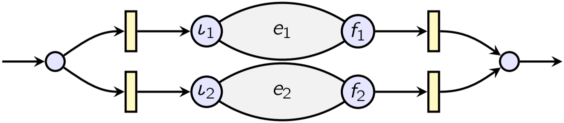
$[\![\mathcal{N}]\!]$ is the set of pomset-traces of accepting runs of $\mathcal{N}$.

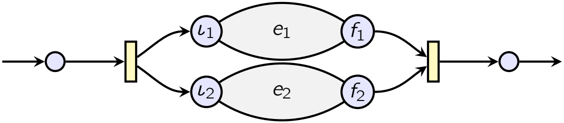
Definition

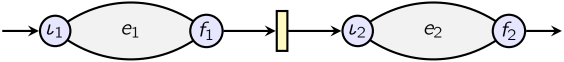
A set of pomsets S is a **recognisable pomset language** if there is a net $\mathcal{N}$ such that $S = [\![\mathcal{N}]\!]$.

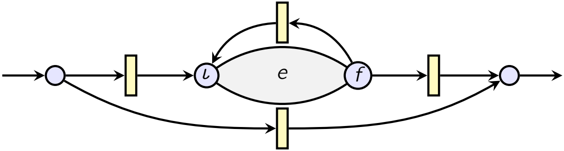
From expressions to automata

$\mathcal{N}(0) :=$  $\mathcal{N}(1) :=$  $\mathcal{N}(a) :=$ 

$\mathcal{N}(e_1 + e_2) :=$ 

$\mathcal{N}(e_1 \parallel e_2) :=$ 

$\mathcal{N}(e_1 \cdot e_2) :=$ 

$\mathcal{N}(e^\star) :=$ 

Solving biKA

Lemma

$$[\![e]\!] = [\![\mathcal{N}(e)]\!].$$

Corollary

Rational pomset languages are recognisable.

Solving biKA

Lemma

$$[\![e]\!] = [\![\mathcal{N}(e)]\!].$$

Corollary

Rational pomset languages are recognisable.

Theorem

Testing containment of pomset-trace languages of two Petri nets is an **ExpSpace**-complete problem.

Jategaonkar & Meyer, **Deciding true concurrency equivalences on safe, finite nets**, *1996*

Corollary

The problem biKA lies in the class **ExpSpace**.

What about CKA?

$${}^{\sqsubseteq}[\![e]\!] = {}^{\sqsubseteq}[\![f]\!]$$

What about CKA?

$${}^{\sqsubseteq}[\![e]\!] = {}^{\sqsubseteq}[\![f]\!] \Leftrightarrow \quad {}^{\sqsubseteq}[\![e]\!] \subseteq {}^{\sqsubseteq}[\![f]\!] \quad \wedge \quad {}^{\sqsubseteq}[\![e]\!] \supseteq {}^{\sqsubseteq}[\![f]\!]$$

What about CKA?

$$
\begin{aligned}
{}^{\sqsubseteq}[\![e]\!] = {}^{\sqsubseteq}[\![f]\!] &\Leftrightarrow \quad {}^{\sqsubseteq}[\![e]\!] \subseteq {}^{\sqsubseteq}[\![f]\!] \quad \wedge \quad {}^{\sqsubseteq}[\![e]\!] \supseteq {}^{\sqsubseteq}[\![f]\!] \\
&\Leftrightarrow \quad [\![e]\!] \subseteq {}^{\sqsubseteq}[\![f]\!] \quad \wedge \quad {}^{\sqsubseteq}[\![e]\!] \supseteq [\![f]\!]
\end{aligned}
$$

What about CKA?

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{}^{\sqsubseteq}[\![e]\!] = {}^{\sqsubseteq}[\![f]\!] &\Leftrightarrow \quad {}^{\sqsubseteq}[\![e]\!] \subseteq {}^{\sqsubseteq}[\![f]\!] \quad \wedge \quad {}^{\sqsubseteq}[\![e]\!] \supseteq {}^{\sqsubseteq}[\![f]\!] \\
&\Leftrightarrow \quad [\![e]\!] \subseteq {}^{\sqsubseteq}[\![f]\!] \quad \wedge \quad {}^{\sqsubseteq}[\![e]\!] \supseteq [\![f]\!] \\
&\Leftrightarrow [\![\mathcal{N}(e)]\!] \subseteq {}^{\sqsubseteq}[\![\mathcal{N}(f)]\!] \wedge {}^{\sqsubseteq}[\![\mathcal{N}(e)]\!] \supseteq [\![\mathcal{N}(f)]\!]
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\end{aligned}$$

Problem

Let $\mathcal{N}_1, \mathcal{N}_2$ be well behaved nets. Is it true that for every run R_1 of $\mathcal{N}_1$ there is a run R_2 in $\mathcal{N}_2$ such that

$$\mathcal{P}om(R_1) \sqsubseteq \mathcal{P}om(R_2)?$$

Idea of the algorithm

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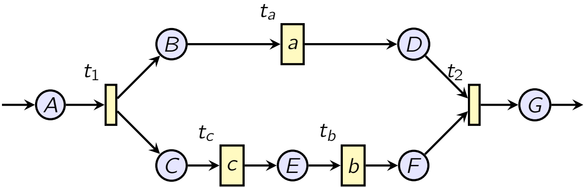
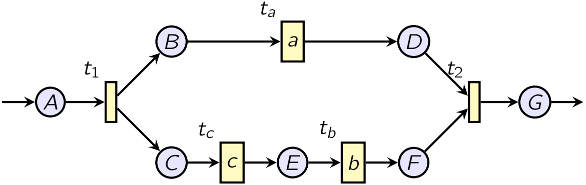
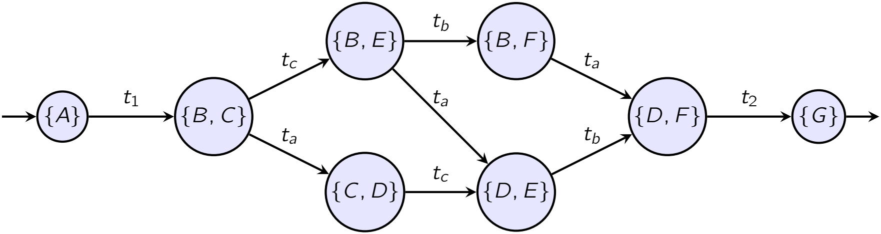
- $[\![\mathcal{N}_1]\!] \subseteq^{\sqsubseteq} [\![\mathcal{N}_2]\!]$ if and only if $\mathcal{L}\,(\mathscr{A}_1) = \mathcal{L}\,(\mathscr{A}_2)$.

Transition automaton

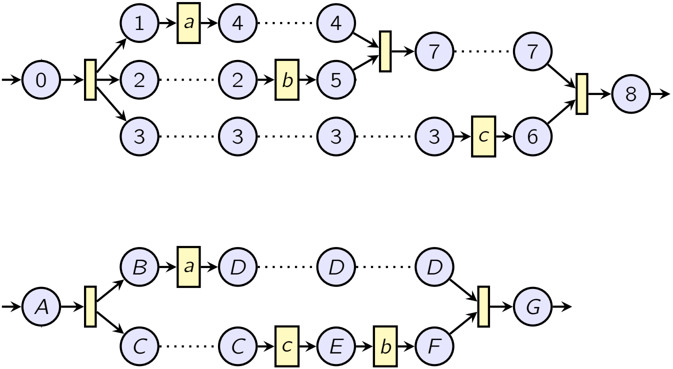


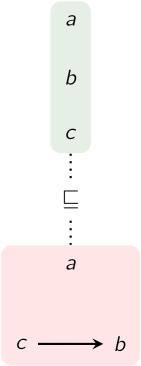
Transition automaton





Massaging runs





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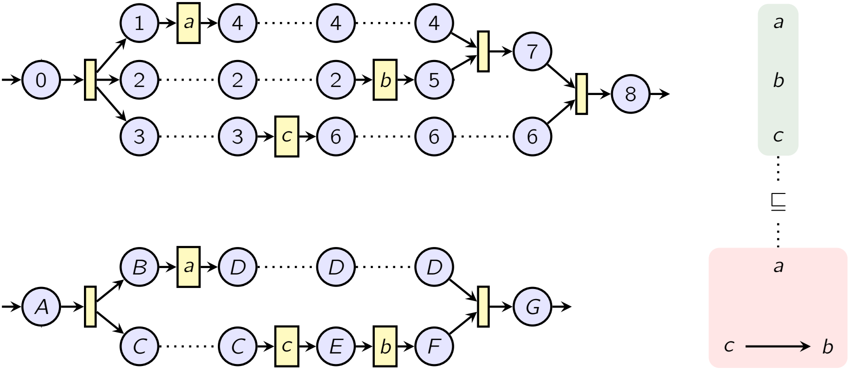

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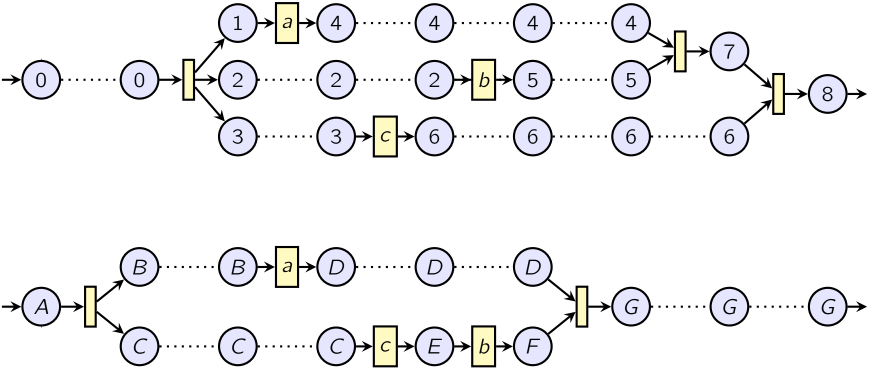

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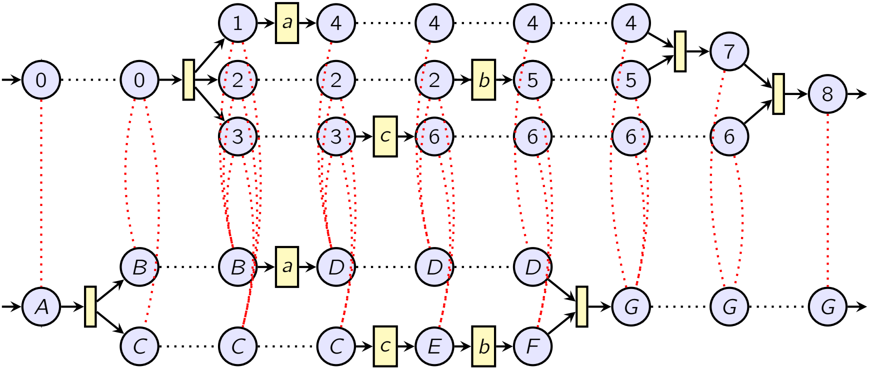

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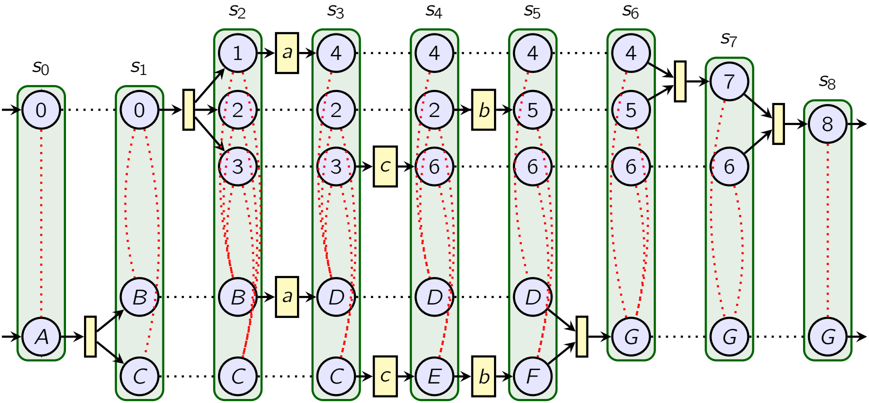

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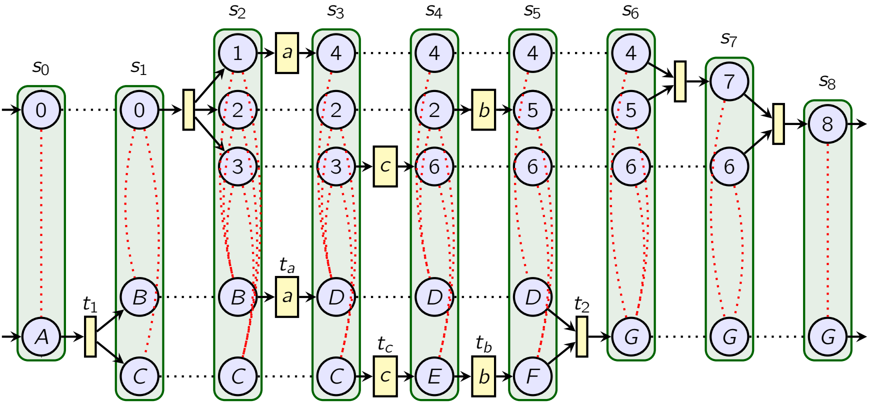

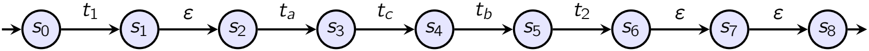

Reduction to automata

Let $\mathcal{N}_1$ and $\mathcal{N}_2$ be some polite nets, of size n, m.

Lemma

There is an automaton $\mathscr{A}(\mathcal{N}_1)$ with $\mathcal{O}(2^n)$ states that recognises the set of accepting runs in $\mathcal{N}_1$.

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Lemma

There is an automaton $\mathcal{N}_1 \prec \mathcal{N}_2$ with $\mathcal{O}(2^{n+m+nm})$ states that recognises the set of accepting runs in $\mathcal{N}_1$ whose pomset belongs to ${}^{\sqsubseteq}[\![\mathcal{N}_2]\!]$.

Outline

I. Pomsets

II. Petri Nets

III. Summary and Outlook

Main result

Theorem

Given two expressions $e, f \in \mathbb{E}_{\Sigma}$, we can test if $[\![e]\!] \subseteq^{\sqsubseteq} [\![f]\!]$ in **ExpSpace**.

Proof.

1. build $\mathcal{N}(e)$ and $\mathcal{N}(f)$;
2. build $\mathscr{A}(\mathcal{N}(e))$ and $\mathcal{N}(e) \prec \mathcal{N}(f)$;
3. compare them.

$\square$

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□

Theorem

The problem CKA is **ExpSpace**-complete.

Proof.

1. In the class **ExpSpace**: see above.
2. **ExpSpace**-hard: Reduction from the universality problem for regular expressions with interleaving.

Mayer & Stockmeyer, **The complexity of word problems – this time with interleaving**, *1994* □

To sum up

Done:

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- Reduction of biKA and CKA to **Petri nets**.

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- Extend the algorithm to a larger class of Petri nets.

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- Add **names** because they're fun!

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- *Insert you favourite operator here...*

That's all folks!

Thank you!

See more at:
`http://paul.brunet-zamansky.fr`

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